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Exact cross sections for the neutralino WIMP pair-annihilation

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Abstract: We derive a full set of exact, analytic expressions for the annihilation of the lightest neutralino pairs into all two-body tree-level final states in the framework of minimal supersymmetry. We make no simplifying assumptions about the neutralino nor about sfermion masses and mixings other than the absence of explicit CP-violating terms. The expressions should be particularly useful in computing the neutralino WIMP relic abundance without the usual approximation of partial wave expansion.

Keywords: Supersymmetric Effective Theories, Cosmology of Theories beyond the SM, Dark Matter.

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1. Introduction

The quest for identifying the nature of the dark matter (DM) in the Universe continues [1, 2]. It is generally believed that most of the DM is made of some hypothetical weakly-interacting massive particles (WIMPs). From the particle theory point of view, a commonly considered candidate for the WIMP is the lightest neutralino under the assumption that it is the lightest supersymmetric particle (LSP). In most approaches the LSP is stable due to an additional R-parity [3]. The neutralino, being massive, often provides a sizeable contribution to the relic density. In addition, the requirement that the neutralino, or some other stable particle relic, does not “overclose” the Universe, often provides a strong constraints on a supersymmetric model. The two other robust candidates for the LSP and cold dark matter (CDM) are the axino (superpartner of the axion) [4] and the gravitino [5].

Continuing improvements in determining the abundance of cold dark matter, and other components of the Universe, have now reached an unprecedented precision of a few per cent [6]. In light of this, one needs to be able to perform an accurate enough computation of the WIMP relic abundance, which would allow for a reliable comparison between theory and observation.

The literature on the relic abundance of the neutralino is vast and still growing. (For a comprehensive review, see ref. [2].) A brief, and by no means complete, account of major developments, can be summarized as follows. The original paper by Goldberg [7] considered the neutralino in the photino limit and pointed out the strong constraints from its relic abundance. This was soon followed the first analysis by Ellis, et al. [8] and Krauss [9], of the general neutralino case. Several other early papers subsequently appeared with more detailed and elaborate analyses. In particular, Griest [10] was first to compute in detail the annihilation into the ordinary fermion-pair ($f\bar{f}$) final states through the Z-exchange, and later Griest, Kamionkowski and Turner [11] conducted the first more complete analysis of the general neutralino case into WW, ZZ and Higgs-pair final states. The Higgs contribution to $f\bar{f}$ was first computed in refs. [12, 11]. Olive and Srednicki [13] considered all the annhilation channels but only in the limit of the pure gaugino and higgsino cases where several important resonances and final states are absent. Drees and Nojiri [14] computed a first complete set of expressions for the product of the cross section times velocity using the helicity amplitude technique. When expanded in the nonrelativistic limit, these give expressions for the first two coefficients of the partial wave expansion.

In the early papers the partial wave expansion of the thermally-averaged product of the neutralino pair-annihilation cross section and their relative velocity, $\langle\sigma v\rangle \approx a + bx$ was used in most cases. The method is normally expected to give an accurate enough approximation (about few per cent) but only far enough from s-channel resonances and thresholds for new final states, as was first pointed out by Griest and Seckel [15] and further emphasized in refs. [16]–[19]. In particular, it was shown in ref. [17] that, because of the very narrow width of the lightest supersymmetric Higgs h, in the vicinity of its s-channel exchange the error can be as large as a few orders of magnitude. Partial remedies were suggested, for example in [17, 20], by numerically integrating the full cross section near resonances only and by matching this with the expansion-based calculations further away from poles. Such methods are not fully satisfactory since they do not include interference terms. A recent detailed analysis [19] showed that in the case of the often wide s-channel resonance caused by the exchange of the pseudoscalar A, the expansion produces a significant error over the range of neutralino mass which can be as big as several tens of GeV. Furthermore subdominant channels and often neglected interference terms can also sometimes play a sizeable role.

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A formalism for computing the relic abundance also became refined. In particular, the effect of replacing the usually assumed common heat bath for both annihilating particles [1] by a more accurate treatment of involving two separate thermal distributions was considered by Gondolo and Gelmini [16] and by Srednicki, et al. [21]. In practical terms the numerical difference is usually negligible when one does the usual partial wave expansion. The first coefficient a is universal while the second ones b differ by $3a/2$, where usually $a \ll b$. Gondolo and Gelmini [16] further derived a very useful compact expression for the thermally-averaged product of the neutralino pair-annihilation cross section and their relative velocity as a single integral over the cross section, as we will see below. Gondolo and collaborators next developed a Fortran code DarkSusy [22] where the relic density of neutralinos is numerically computed without using the partial wave expansion approximation.

An additional effect of reducing the relic abundance of WIMPs through coannihilation was first pointed out by Griest and Seckel [15]. In some cases there may exist some other states which are not much heavier than the stable WIMP and may therefore be still present in the thermal plasma around the WIMP decoupling. In the framework of minimal supersymmetry with the lightest neutralino LSP with a significant higgsino component, the coannihilation with the next-to-lightest neutralino and the lightest chargino is often important [23, 24]. Other cases of interest involve neutralino annihilation with the lighter superpartner of the τ-lepton [25] and with lighter stop [26].

In this paper, we will present a full set of exact analytic expressions for the cross section of the neutralino pair-annihilation in the general Minimal Supersymmetric Standard Model (MSSM) for a general neutralino case. From the point of view of low-energy supersymmetry, the most natural choice for the LSP and CDM is a nearly pure gaugino (bino) as was first shown by Roszkowski [27]. Remarkably, just such a case of the [electrically neutral] LSP naturally emerges in most case in the Constrained MSSM [28, 17, 29]. Nevertheless, in our analysis we will make *no* simplifying assumptions about the neutralino, nor will we assume the degeneracy of the left- and right-sfermion masses. We will include all tree-level final states and all intermediate states. We will also keep finite widths in s-channel resonances. We will only neglect possible CP-violating phases in the SUSY sector. We will also not consider the effect of coannihilation here but will address it in a subsequent publication. A complete set of expressions presented here does not rely on the partial wave expansion, includes all the terms and is valid both near and further away from resonances and thresholds for new final states.

Some of the results presented here are not new but we include them here nevertheless in order to provide a complete and self-consistent reference containing the full set of exact expressions. In particular, the cross sections for the neutralino pair-annihilation into the SM fermion-pair ($f\bar{f}$) final states were first computed in ref [10] and for the WW, ZZ and Higgs boson-pair final states in ref. [11].

Given the complexity of analytic expressions presented here and in other papers, and due to often different conventions used, it is not doable to perform a real comparison with the literature. Instead, we have performed a numerical check of our cross section with the results obtained by using DarkSusy [22]. We have found, for the same values of input parameters, an impressive agreement, at the level of a few per cent, for all the annihilation channels, which we find reassuring. (Recently another numerical code has made available [30] and also numerically agreed with DarkSusy.)

While the exact analytic expressions presented here are applicable both near and away from special cases where the method of partial wave expansion fails, sometimes one may find it less CPU-time consuming to use the latter one. Starting from our exact cross sections we have therefore derived a complete set of expressions for the usual first two terms of expansion for all the dominant channels. We will present them here as well.

The annihilation into $f\bar{f}$ is often dominant. However, other final states can also play important role, depending on the case. In a previous paper [19] we performed a detailed numerical comparison of the relic abundance computed using the exact formulae with the one obtained using the expansion formulae, for all the channels, including subdominant ones. Our analysis confirmed that the expansion gives highly inaccurate results near resonances and new thresholds. We also showed that very far from such cases the error is typically rather small, of the order of a few per cent. However, we found that, because of the existence of several resonances (Z and the Higgs bosons), the expansion produces large errors, compared to an exact treatment, over a sizeable range of the neutralino mass m_χ, even of a several tens of Gev. In other words, the widely used method of expansion may lead to significant errors in a sizeable fraction of the neutralino mass.

The plan of the paper is as follows. In section 2 we review the formalism for computing the relic density that we employ. In section 3 we introduce the relevant ingredients of the MSSM and list all the neutralino pair-annihilation channels. Explicit expressions for the annihilation cross secion are given in section 4. In section 5 we discuss expansion and provide a list of formulae for the first two coefficients in the case of equal-mass final states. In section 6 we summarize our work. Appendix A contains a complete list of lagrangian terms and couplings which are used in the paper while in appendix B we provide expressions for several auxiliary functions used in the text.

2. Calculation of the relic density

The relic abundance of some stable species χ is defined as $\Omega_\chi \equiv \rho_\chi/\rho_{\rm crit}$, where $\rho_\chi = m_\chi n_\chi$ is the relic's mass density, n_χ is its number density, $\rho_{\rm crit} \equiv 3H_0^2/8\pi G_N = 1.9 \times 10^{-29}\,(h^2)\,{\rm g/cm}^3$ is the critical density and G_N is the gravitational constant. (For a review of relic density calculations, see, e.g., refs. [1, 2].) The time evolution and subsequent freeze-out of n_χ in an expanding Universe are described by the Boltzmann equation

$$\frac{dn_\chi}{dt} = -3Hn_\chi - \langle \sigma v_{\rm M\o l}\rangle \left[n_\chi^2 - (n_\chi^{\rm eq})^2\right], \tag{2.1}$$

where $n_\chi^{\rm eq}$ is the number density that the species would have in thermal equilibrium, $H(T)$ is the Hubble expansion rate, $\sigma(\chi\chi \to \text{all})$ denotes the cross section of the species annihilation into ordinary particles, $v_{\rm M\o l}$ is a so-called Møller velocity [16] which is the relative velocity of the annihilating particles, and $\langle \sigma v_{\rm M\o l}\rangle$ represents the thermal average of $\sigma v_{\rm M\o l}$ which will be given below. In the early Universe, the species χ were initially in thermal equilibrium,

$n_\chi = n_\chi^{\rm eq}$. When their typical interaction rate Γ_χ became less than the Hubble parameter, $\Gamma_\chi \lesssim H$, the annihilation process froze out. Since then their number density in a co-moving volume has remained basically constant.

The thermally-averaged product of the neutralino pair-annihilation cross section and their relative velocity $\langle \sigma v_{\rm Møl} \rangle$ is most properly defined in terms of separate thermal baths for both annihilating particles [16, 21]

$$\langle \sigma v_{\rm Møl} \rangle (T) = \frac{\int d^3p_1 d^3p_2\, \sigma v_{\rm Møl}\, e^{-E_1/T} e^{-E_2/T}}{\int d^3p_1 d^3p_2\, e^{-E_1/T} e^{-E_2/T}}\,, \tag{2.2}$$

where $p_1 = (E_1, \mathbf{p}_1)$ and $p_2 = (E_2, \mathbf{p}_2)$ are the 4-momenta of the two colliding particles, and T is the temperature of the bath. (Note that one often uses another definition of $\langle \sigma v_{\rm Møl} \rangle$ which involves a single thermal bath for both neutralinos. Compare, e.g., refs. [1, 14, 2]. The numerical difference between the two formulae is usually rather small.) The above expression can be reduced to a one-dimensional integral which can be written in a Lorentz-invariant form as [16]

$$\langle \sigma v_{\rm Møl} \rangle (T) = \frac{1}{8 m_\chi^4 T K_2^2(m_\chi/T)} \int_{4m_\chi^2}^{\infty} ds\, \sigma(s)(s - 4m_\chi^2)\sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right)\,, \tag{2.3}$$

where $s = (p_1 + p_2)^2$ is a usual Mandelstam variable and K_i denotes the modified Bessel function of order i. In computing the relic abundance one first evaluates eq. (2.3) and then uses this to solve the Boltzmann eq. (2.1).

There are a number of methods of solving eq. (2.1). One often used, approximate, although in general quite accurate (for a recent discussion see ref. [19]), solution to the Boltzmann equation is based on solving iteratively the equation

$$x_f^{-1} = \ln\left(\frac{m_\chi}{2\pi^3}\sqrt{\frac{45}{2 g_* G_N}} \langle \sigma v_{\rm Møl} \rangle (x_f)\, x_f^{1/2}\right)\,, \tag{2.4}$$

where g_* represents the effective number of degrees of freedom at freeze-out ($\sqrt{g_*} \simeq 9$). Typically one finds that the freeze-out point $x_f \equiv T_f/m_\chi$ is roughly given by 1/25-1/20. One usually introduces $J(x_f)$ defined as

$$J(x_f) \equiv \int_0^{x_f} dx \langle \sigma v_{\rm Møl} \rangle (x)\,, \tag{2.5}$$

where $x = T/m_\chi$.

The relic density at present is given by

$$\rho_\chi = \frac{1.66}{M_{\rm Pl}} \left(\frac{T_\chi}{T_\gamma}\right)^3 T_\gamma^3 \sqrt{g_*} \frac{1}{J(x_f)}\,, \tag{2.6}$$

where $M_{\rm Pl} = 1/\sqrt{G_N}$ denotes the Planck mass, T_χ and T_γ are the present temperatures of the neutralino and the photon, respectively. The suppression factor $(T_\chi/T_\gamma)^3 \approx 1/20$ follows from entropy conservation in a comoving volume [31].

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3. WIMP annihilation in the MSSM

In this section we introduce the relevant parameters and definitions. We will be working in the framework of the general MSSM. (For a review, see, e.g., ref. [3]. We follow the conventions of ref. [32].) The lightest neutralino is a mass eigenstate given by a linear combination of the bino $\widetilde{B}$, the neutral wino $\widetilde{W}_3^0$ and the two neutral higgsinos $\widetilde{H}_b^0$ and $\widetilde{H}_t^0$

$$\chi \equiv \chi_1^0 = N_{11}\widetilde{B} + N_{12}\widetilde{W}_3^0 + N_{13}\widetilde{H}_b^0 + N_{14}\widetilde{H}_t^0 \,. \tag{3.1}$$

The neutralino mass matrix is determined by the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ gaugino mass parameters M_1 and M_2, respectively (and we impose the usual GUT relation $M_1 = \frac{5}{3}\tan^2\theta_W M_2$), the Higgs/higgsino mass parameter μ, the usual weak angle θ_W and $\tan\beta = v_t/v_b$ — the ratio of the vacuum expectation values of the two neutral Higgs fields.

The neutralino mass matrix is given by

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_1 & 0 & -m_Z\sin\theta_W\cos\beta & m_Z\sin\theta_W\sin\beta \\ 0 & M_2 & m_Z\cos\theta_W\cos\beta & -m_Z\cos\theta_W\sin\beta \\ -m_Z\sin\theta_W\cos\beta & m_Z\cos\theta_W\cos\beta & 0 & -\mu \\ m_Z\sin\theta_W\sin\beta & -m_Z\cos\theta_W\sin\beta & -\mu & 0 \end{pmatrix} . \tag{3.2}$$

The neutralino mass matrix is diagonalized by a unitary matrix N

$$N^*\mathcal{M}_{\chi^0}N^{-1} = \mathrm{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}) \,. \tag{3.3}$$

In the absence of possible CP violating phases, one can choose a basis such that the mixing matrix N is real, in which case some of the neutralino masses will in general be negative.

The chargino mass matrix is given by

$$\mathcal{M}_{\chi^\pm} = \begin{pmatrix} M_2 & \sqrt{2}m_W\sin\beta \\ \sqrt{2}m_W\cos\beta & \mu \end{pmatrix} . \tag{3.4}$$

The chargino mass matrix is diagonalized by two unitary matrices U and V

$$U^*\mathcal{M}_{\chi^\pm}V^{-1} = \mathrm{diag}(m_{\chi_1^\pm}, m_{\chi_2^\pm}) \,. \tag{3.5}$$

There are two neutral scalar Higgs bosons h and H, a pseudoscalar A plus a pair of charged Higgs $H^\pm$. (We will typically suppress the Higgs charge assignment except where this may lead to ambiguities.)

Other relevant parameters which determine the masses of scalars and various couplings are the squark soft mass parameters m_Q, m_U and m_D, the slepton soft mass parameters m_L and m_E, and the pseudoscalar mass m_A. We also include the trilinear terms A_i ($i = t, b, \tau$) of the third generation which are important in determining the masses and couplings for the stop $\widetilde{t}_{1,2}$, sbottom $\widetilde{b}_{1,2}$ and stau $\widetilde{\tau}_{1,2}$ states, respectively.

In general the flavor-violating sfermion mass-squared (6×6) matrices are given by

$$\mathcal{M}^2_{\widetilde{f}} = \begin{pmatrix} m^2_{LL}(\widetilde{f}) & m^2_{LR}(\widetilde{f}) \\ m^2_{RL}(\widetilde{f}) & m^2_{RR}(\widetilde{f}) \end{pmatrix} , \tag{3.6}$$

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with

$$
\begin{aligned}
m^2_{LL}(\widetilde{f}) &= M^2_{\widetilde{f}L} + M_f^\dagger M_f + m_Z^2 \cos 2\beta \left(T_{3f} - Q_f \sin^2\theta_W\right) \mathbf{1}\,, && (3.7)\\
m^2_{RR}(\widetilde{f}) &= M^2_{\widetilde{f}R} + M_f M_f^\dagger + m_Z^2 \cos 2\beta\, Q_f \sin^2\theta_W \mathbf{1}, && (3.8)\\
m^2_{LR}(\widetilde{f}) &= \begin{cases} M_f^\dagger \left(A_f^\dagger - \mu \cot\beta\right) & \text{for } T_{3f} = +1/2 \\ M_f^\dagger \left(A_f^\dagger - \mu \tan\beta\right) & \text{for } T_{3f} = -1/2 \end{cases}\,, && (3.9)\\
m^2_{RL}(\widetilde{f}) &= m^{2\,\dagger}_{LR}(\widetilde{f})\,, && (3.10)
\end{aligned}
$$

where $\widetilde{f}$ and f denote here different types of (s)fermions (up- and down-type (s)quarks, charged and neutral (s)leptons). For (s)neutrinos only the LL part of eq. (3.6) should be taken. $M^2_{\widetilde{f}L}$ and $M^2_{\widetilde{f}R}$ denote 3×3 soft SUSY-breaking sfermion mass matrices, and M_f denotes here a 3×3 fermion mass matrix. Finally, A_f is a scalar trilinear coupling matrix of the same dimension while Q_f and T_{3f} are the respective electric and isospin charges. All the interaction terms and couplings that we will need below are summarized in appendix A.

In the MSSM, the neutralino LSP's can pair-annihilate into a number of final states, if kinematically allowed. A complete list of all tree-level two-body final states is given in table 1. We only neglect two-body loop processes into final state photon pairs and gluon pairs because they are always subdominant in computing the relic density [2]. We also neglect three-body final states since they are unlikely to be competitive with two-body final states. They were shown to dominate in the higgsino case just below the WW and $t\bar{t}$ final states [33] but in such regions neutralino coannihilation with the lightest chargino and next-to-lightest neutralino reduce the relic density to very small values anyway.

For the channels WW, ZZ, $\bar{t}t$, $W^\pm H^\mp$, Zh, ZH, Ah and AH the s-wave is not suppressed, and, once kinematically allowed, these can give dominant contributions. But even the s-wave suppressed channels $\bar{f}f$ ($f \neq t$), hh, Hh, HH, AA, H^+H^- and ZA can play some role, especially if the other channels are not yet kinematically allowed. This in particular is the case with the light fermion-pair final states for which the cross section is suppressed by the square of the corresponding fermion mass but which are always kinematically allowed and often dominant.

4. Exact expressions

We now proceed to present a full set of exact, analytic expressions for the total cross section $\sigma(\chi\chi \to \text{all})$ for the neutralino pair-annihilation processes into all allowed (tree-level) two-body final states in the general MSSM. We have included all contributing diagrams as well as all interference terms and kept finite widths of all s-channel resonances. We have made no simplifying assumptions about sfermion masses although we assumed that there are no CP violating phases in SUSY parameters.

To start with, it is convenient to introduce a Lorentz-invariant function $w(s)$ [21]

$$
w(s) = \frac{1}{4} \int d\,\mathrm{LIPS}\, |\mathcal{A}(\chi\chi \to \text{all})|^2\,, \qquad (4.1)
$$

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Process	Exchanged particles	
	s-channel	t- and u-channel
$\chi\chi \to hh$	h, H	χ_i^0
$\chi\chi \to HH$	h, H	χ_i^0
$\chi\chi \to hH$	h, H	χ_i^0
$\chi\chi \to AA$	h, H	χ_i^0
$\chi\chi \to hA$	A, Z	χ_i^0
$\chi\chi \to HA$	h, H	χ_i^0
$\chi\chi \to H^+H^-$	h, H, Z	$\chi_k^\pm$
$\chi\chi \to W^\pm H^\mp$	h, H, A	$\chi_k^\pm$
$\chi\chi \to Zh$	A, Z	χ_i^0
$\chi\chi \to ZH$	A, Z	χ_i^0
$\chi\chi \to ZA$	h, H	χ_i^0
$\chi\chi \to W^+W^-$	h, H, Z	$\chi_k^\pm$
$\chi\chi \to ZZ$	h, H	χ_i^0
$\chi\chi \to f\bar{f}$	h, H, A, Z	$\widetilde{f}_a$

Table 1: A complete set of neutralino pair-annihilation channels into tree-level two-body final states in the MSSM. The indices i, k, a run as follows: $i = 1, \ldots, 4$, $k = 1, 2$ and $a = 1, \ldots, 6$.

where $|\mathcal{A}(\chi\chi \to \text{all})|^2$ denotes the absolute square of the reduced matrix element for the annihilation of two χ particles, averaged over initial spins and summed over final spins. The function $w(s)$ is related to the annihilation cross section $\sigma(s)$ in eq. (2.3) via [18]

$$w(s) = \frac{1}{2}\sqrt{s(s-4m_\chi^2)}\,\sigma(s)\,. \tag{4.2}$$

Since $w(s)$ receives contributions from all the kinematically allowed annihilation process $\chi\chi \to f_1 f_2$, it can be written as

$$w(s) = \frac{1}{32\,\pi}\sum_{f_1 f_2}\left[c\,\theta\left(s-(m_{f_1}+m_{f_2})^2\right)\beta_f(s, m_{f_1}, m_{f_2})\,\widetilde{w}_{f_1 f_2}(s)\right], \tag{4.3}$$

where the summation extends over all possible two-body final states $f_1 f_2$, m_{f_1} and m_{f_2} denote their respective masses, and

$$c = \begin{cases} c_f \text{ if } f_{1(2)} = f(\bar{f}) \\ 1 \quad \text{otherwise}\,, \end{cases} \tag{4.4}$$

where c_f is the color factor of SM fermions ($c_f = 3$ for quarks and $c_f = 1$ for leptons). The kinematic factor β_f is defined as

$$\beta_f(s, m_{f_1}, m_{f_2}) \equiv \left[1-\frac{(m_{f_1}+m_{f_2})^2}{s}\right]^{1/2}\left[1-\frac{(m_{f_1}-m_{f_2})^2}{s}\right]^{1/2}. \tag{4.5}$$

In the CM frame, which we choose for convenience, the function $\widetilde{w}_{f_1 f_2}(s)$ can be expressed as

$$\widetilde{w}_{f_1 f_2}(s) \equiv \frac{1}{2}\int_{-1}^{+1} d\cos\theta_{CM}\,|\mathcal{A}(\chi\chi \to f_1 f_2)|^2\,, \tag{4.6}$$

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where θ_{CM} denotes the scattering angle in the CM frame. In other words, we write $|\mathcal{A}(\chi\chi \to f_1 f_2)|^2$ as a function of s and $\cos\theta_{CM}$, which greatly simplifies the computation.

We will follow table 1 in presenting explicit expressions for $\widetilde{w}_{f_1 f_2}(s)$ for all the two-body final states. All the couplings are defined in appendix A. All other auxiliary functions, are listed in appendix B. Feynman diagrams corresponding to all the annihilation channels are given in ref. [2, chapter 6].

4.1 $\chi\chi \to hH$

This process involves the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{w}_{hH} = \widetilde{w}_{hH}^{(h,H)} + \widetilde{w}_{hH}^{(\chi^0)} + \widetilde{w}_{hH}^{(h,H-\chi^0)} \; : \tag{4.7}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{w}_{hH}^{(h,H)} = \frac{1}{2} \left| \sum_{r=h,H} \frac{C^{hHr}\, C_S^{\chi\chi r}}{s - m_r^2 + i\,\Gamma_r\, m_r} \right|^2 (s - 4\, m_\chi^2)\, ; \tag{4.8}$$

Neutralino (χ_i^0) exchange:

$$\widetilde{w}_{hH}^{(\chi^0)} = \sum_{i,j=1}^{4} C_S^{\chi_i^0 \chi h}\, (C_S^{\chi_j^0 \chi h})^*\, C_S^{\chi_i^0 \chi H}\, (C_S^{\chi_j^0 \chi H})^* \left[m_{\chi_i^0}\, m_{\chi_j^0} I_{ij}^{hH} + m_\chi m_{\chi_i^0} J_{ij}^{hH} + K_{ij}^{hH} \right], \tag{4.9}$$

where

$$I_{ij}^{hH} = (s - 4\, m_\chi^2) \Big[\mathcal{T}_0 - \mathcal{Y}_0 \Big] (s, m_\chi^2, m_h^2, m_H^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\, , \tag{4.10}$$

$$\begin{aligned} J_{ij}^{hH} = & \Big[-4\, \mathcal{T}_1 + 2(-2\, m_\chi^2 + m_h^2 + m_H^2) \mathcal{T}_0 - 2\, \mathcal{Y}_1 - \\ & - 2(s - 4\, m_\chi^2) \mathcal{Y}_0 \Big] (s, m_\chi^2, m_h^2, m_H^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\, , \end{aligned} \tag{4.11}$$

$$\begin{aligned} K_{ij}^{hH} = & \Big\{ -\mathcal{T}_2 - (s + 2\, m_\chi^2 - m_h^2 - m_H^2) \mathcal{T}_1 - (m_\chi^2 - m_h^2)(m_\chi^2 - m_H^2) \mathcal{T}_0 - \\ & - \mathcal{Y}_2 + [-(m_\chi^2 + m_h^2)(m_\chi^2 + m_H^2) + 4\, m_\chi^4] \mathcal{Y}_0 \Big\} (s, m_\chi^2, m_h^2, m_H^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\, ; \end{aligned} \tag{4.12}$$

Higgs (h, H)-neutralino (χ_i^0) interference term:

$$\begin{aligned} \widetilde{w}_{hH}^{(h,H-\chi^0)} = & \; 2 \sum_{i=1}^{4} \mathrm{Re} \left[\sum_{r=h,H} \left(\frac{C^{hHr}\, C_S^{\chi\chi r}}{s - m_r^2 + i\,\Gamma_r\, m_r} \right)^* C_S^{\chi_i^0 \chi h} C_S^{\chi_i^0 \chi H} \right] \times \\ & \times \Big\{ -2\, m_\chi + [m_\chi\, (m_H^2 + m_h^2 - 2\, m_\chi^2 - 2\, m_{\chi_i^0}^2) + \\ & \qquad + m_{\chi_i^0}\, (s - 4\, m_\chi^2)] \mathcal{F}(s, m_\chi^2, m_H^2, m_h^2, m_{\chi_i^0}^2) \Big\}\, . \end{aligned} \tag{4.13}$$

As mentioned above, the couplings $C_S^{\chi_i^0 \chi_j^0 r}$ and C^{hHr} ($r = h, H$), as well as all the other couplings appearing in this section, are defined in appendix A. The functions $\mathcal{F}$, $\mathcal{T}_k$ and

$\mathcal{Y}_k$ ($k = 0, \ldots, 4$), and all other auxiliary functions, are listed in appendix B. By Γ_h and Γ_H we denote the widths of h and H, respectively.

The expressions for hh final state are obtained from the above by replacing m_H, C^{hHr}, $C_S^{\chi_i^0\chi H}$, $C_S^{\chi_j^0\chi H}$ with m_h, C^{hhr}, $C_S^{\chi_i^0\chi h}$, $C_S^{\chi_j^0\chi h}$, respectively, and multiplying ω by a factor of 1/2 for identical particles in the final state. The contributions for HH final state are obtained in an analogous way.

4.2 $\chi\chi \to AA$

Similarly to the final state hH, this process proceeds via the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{w}_{AA} = \widetilde{w}_{AA}^{(h,H)} + \widetilde{w}_{AA}^{(\chi^0)} + \widetilde{w}_{AA}^{(h,H-\chi^0)} \; : \tag{4.14}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{w}_{AA}^{(h,H)} = \frac{1}{4}\left|\sum_{r=h,H}\frac{C^{AAr}\,C_S^{\chi\chi r}}{s - m_r^2 + i\,\Gamma_r\,m_r}\right|^2 (s - 4\,m_\chi^2)\,; \tag{4.15}$$

Neutralino (χ_i^0) exchange:

$$\widetilde{w}_{AA}^{(\chi^0)} = \frac{1}{2}\sum_{i,j=1}^{4}\left(C_P^{\chi_i^0\chi A}\right)^2\left(C_P^{\chi_j^0\chi A*}\right)^2\Big[m_{\chi_i^0}\,m_{\chi_j^0}I_{ij}^{AA} + m_\chi m_{\chi_i^0}J_{ij}^{AA} + K_{ij}^{AA}\Big], \tag{4.16}$$

where

$$I_{ij}^{AA} = (s - 4\,m_\chi^2)\Big[\mathcal{T}_0 - Y_0\Big](s, m_\chi^2, m_A^2, m_A^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \tag{4.17}$$

$$J_{ij}^{AA} = \Big[4\,\mathcal{T}_1 + 4(m_\chi^2 - m_A^2)\mathcal{T}_0 + 2\,\mathcal{Y}_1 + 2(s - 4\,m_\chi^2)\mathcal{Y}_0\Big](s, m_\chi^2, m_A^2, m_A^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \tag{4.18}$$

$$\begin{aligned} K_{ij}^{AA} = &\Big\{-T_2 - [s + 2\,(m_\chi^2 - m_A^2)]\mathcal{T}_1 - (m_\chi^2 - m_A^2)^2\mathcal{T}_0 - \\ &- \mathcal{Y}_2 + [4\,m_\chi^4 - (m_\chi^2 + m_A^2)^2]\mathcal{Y}_0\Big\}(s, m_\chi^2, m_A^2, m_A^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,; \end{aligned} \tag{4.19}$$

Higgs (h, H)-neutralino (χ_i^0) interference term:

$$\begin{aligned} \widetilde{w}_{AA}^{(h,H-\chi^0)} = &-\sum_{i=1}^{4}\mathrm{Re}\left[\sum_{r=h,H}\left(\frac{C^{AAr}\,C_S^{\chi\chi r}}{s - m_r^2 + i\,\Gamma_r\,m_r}\right)^* (C_P^{\chi_i^0\chi A})^2\right] \times \\ &\times\Big\{-2\,m_\chi + \Big[2\,m_\chi\,(m_A^2 - m_\chi^2 - m_{\chi_i^0}^2) - \\ &- m_{\chi_i^0}\,(s - 4\,m_\chi^2)\Big]\mathcal{F}(s, m_\chi^2, m_A^2, m_A^2, m_{\chi_i^0}^2)\Big\}\,. \end{aligned} \tag{4.20}$$

4.3 $\chi\chi \to hA$

This process proceeds via the s-channel Z and CP-odd Higgs boson (A) exchange as well as the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{w}_{hA} = \widetilde{w}_{hA}^{(A)} + \widetilde{w}_{hA}^{(Z)} + \widetilde{w}_{hA}^{(\chi^0)} + \widetilde{w}_{hA}^{(A-Z)} + \widetilde{w}_{hA}^{(A-\chi^0)} + \widetilde{w}_{hA}^{(Z-\chi^0)} \; : \tag{4.21}$$

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CP-odd Higgs-boson (A) exchange:

$$\widetilde{w}_{hA}^{(A)} = \frac{1}{2} \left| \frac{C^{hAA}\, C_P^{\chi\chi A}}{s - m_A^2 + i\,\Gamma_A\, m_A} \right|^2 s\,; \tag{4.22}$$

Z-boson exchange:

$$\begin{aligned} \widetilde{w}_{hA}^{(Z)} &= \frac{1}{3} \left| \frac{C^{hAZ}\, C_A^{\chi\chi Z}}{s - m_Z^2 + i\,\Gamma_Z\, m_Z} \right|^2 \frac{1}{m_Z^4\, s} \times \\ &\quad \times \Big\{ m_Z^4\, s^3 + s^2 [6\, m_\chi^2\, (m_A^2 - m_h^2)^2 - 4\, m_\chi^2\, m_Z^4 - 2\, m_Z^4\, (m_A^2 + m_h^2)] + \\ &\qquad + s\, m_Z^2 \Big[-12\, m_\chi^2\, (m_A^2 - m_h^2)^2 + 8\, m_Z^2\, m_\chi^2\, (m_A^2 + m_h^2) + \\ &\qquad\qquad + m_Z^2\, (m_A^2 - m_h^2)^2 \Big] + 2\, m_Z^4\, m_\chi^2\, (m_A^2 - m_h^2)^2 \Big\}; \end{aligned} \tag{4.23}$$

Neutralino (χ_i^0) exchange:

$$\widetilde{w}_{hA}^{(\chi^0)} = \sum_{i,j=1}^{4} C_S^{\chi_i^0 \chi h}\, C_P^{\chi_i^0 \chi A} \left(C_S^{\chi_j^0 \chi h} \right)^* \left(C_P^{\chi_j^0 \chi A} \right)^* \Big[m_{\chi_i^0}\, m_{\chi_j^0} I_{ij}^{hA} + m_\chi m_{\chi_i^0} J_{ij}^{hA} + K_{ij}^{hA} \Big], \tag{4.24}$$

where

$$I_{ij}^{hA} = s \Big[\mathcal{T}_0 - \mathcal{Y}_0 \Big] (s, m_\chi^2, m_h^2, m_A^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \tag{4.25}$$

$$J_{ij}^{hA} = 2(m_A^2 - m_h^2) \Big[\mathcal{T}_0 - \mathcal{Y}_0 \Big] (s, m_\chi^2, m_h^2, m_A^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \tag{4.26}$$

$$\begin{aligned} K_{ij}^{hA} &= \Big\{ -T_2 + (2\, m_\chi^2 + m_A^2 + m_h^2 - s) \mathcal{T}_1 + \mathcal{Y}_2 + \\ &\quad + \{ m_\chi^2 (m_A^2 + m_h^2 - m_\chi^2) - m_A^2 m_h^2 \} \Big[\mathcal{T}_0 - \mathcal{Y}_0 \Big] \Big\} (s, m_\chi^2, m_h^2, m_A^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,; \end{aligned} \tag{4.27}$$

Higgs (A)-Z interference term:

$$\begin{aligned} \widetilde{w}_{hA}^{(A-Z)} &= 2\, \mathrm{Re} \left[\left(\frac{C^{hAA} C_P^{\chi\chi A}}{s - m_A^2 + i\,\Gamma_A\, m_A} \right)^* \frac{C^{hAZ}\, C_A^{\chi\chi Z}}{s - m_Z^2 + i\,\Gamma_Z\, m_Z} \right] \times \\ &\quad \times \frac{m_\chi\, (s - m_Z^2)\, (m_A^2 - m_h^2)}{m_Z^2}\,; \end{aligned} \tag{4.28}$$

Higgs (A)-neutralino (χ_i^0) interference term:

$$\begin{aligned} \widetilde{w}_{hA}^{(A-\chi^0)} &= 2 \sum_{i=1}^{4} \mathrm{Re} \left[\left(\frac{C^{hAA} C_P^{\chi\chi A}}{s - m_A^2 + i\,\Gamma_A\, m_A} \right)^* C_S^{\chi_i^0 \chi h}\, C_P^{\chi_i^0 \chi A} \right] \times \\ &\quad \times \Big[s\, m_{\chi_i^0} + m_\chi\, (m_A^2 - m_h^2) \Big]\, \mathcal{F}(s, m_\chi^2, m_h^2, m_A^2, m_{\chi_i^0}^2)\,; \end{aligned} \tag{4.29}$$

Z-neutralino (χ_i^0) interference term:

$$\begin{aligned}\widetilde{w}_{hA}^{(Z-\chi^0)} &= 2\sum_{i=1}^{4} \mathrm{Re}\left[\left(\frac{C^{hAZ}\, C_A^{\chi\chi Z}}{s-m_Z^2+i\,\Gamma_Z\, m_Z}\right)^{*} C_S^{\chi_i^0\chi h}\, C_P^{\chi_i^0\chi A}\right]\times\\ &\quad\times\Bigg[(s+2\,m_{\chi_i^0}^2-2\,m_\chi^2-m_A^2-m_h^2)\,m_Z^2+\\ &\qquad+2\Big\{m_{\chi_i^0}\, s\,[m_Z^2\, m_{\chi_i^0}+m_\chi\,(m_A^2-m_h^2)]-m_Z^2\, m_\chi\, m_{\chi_i^0}\,(m_A^2-m_h^2)+\\ &\qquad\quad+m_Z^2\, m_A^2\, m_h^2-m_Z^2\,(m_A^2+m_h^2)\,(m_{\chi_i^0}^2+m_\chi^2)+m_Z^2\,(m_{\chi_i^0}^2-m_\chi^2)^2+\\ &\qquad\quad+m_\chi^2\,(m_A^2-m_h^2)^2\Big\}\mathcal{F}(s,m_\chi^2,m_h^2,m_A^2,m_{\chi_i^0}^2)\Bigg]. \end{aligned} \tag{4.30}$$

The expressions for HA final state are obtained by replacing m_h, $C_S^{\chi_i^0\chi h}$ and C^{hAZ} in the above with m_H, $C_S^{\chi_i^0\chi H}$ and C^{HAZ}, respectively.

4.4 $\chi\chi\to H^+H^-$

This process proceeds via the s-channel Z and CP-even Higgs boson (h,H) exchange as well as the t- and u-channel chargino ($\chi_k^\pm$, $k=1,2$) exchange

$$\widetilde{w}_{H^+H^-} = \widetilde{w}_{H^+H^-}^{(h,H)} + \widetilde{w}_{H^+H^-}^{(Z)} + \widetilde{w}_{H^+H^-}^{(\chi^\pm)} + \widetilde{w}_{H^+H^-}^{(h,H-\chi^\pm)} + \widetilde{w}_{H^+H^-}^{(Z-\chi^\pm)} \; : \tag{4.31}$$

CP-even Higgs-boson (h,H) exchange:

$$\widetilde{w}_{H^+H^-}^{(h,H)} = \frac{1}{2}\left|\sum_{r=h,H}\frac{C^{H^+H^-r}\, C_S^{\chi\chi r}}{s-m_r^2+i\,\Gamma_r\, m_r}\right|^2 (s-4\,m_\chi^2)\,; \tag{4.32}$$

Z-boson exchange:

$$\widetilde{w}_{H^+H^-}^{(Z)} = \frac{1}{3}\left|\frac{C^{H^+H^-Z}\, C_A^{\chi\chi Z}}{s-m_Z^2+i\,\Gamma_Z\, m_Z}\right|^2 (s-4\,m_\chi^2)\,(s-4\,m_{H^\pm}^2)\,; \tag{4.33}$$

Chargino ($\chi_k^\pm$) exchange:

$$\widetilde{w}_{H^+H^-}^{(\chi^\pm)} = \sum_{k,l=1}^{2}\left[m_{\chi_k^\pm}\, m_{\chi_l^\pm} I_{kl}^{H^+H^-} + m_\chi m_{\chi_k^\pm} J_{kl}^{H^+H^-} + K_{kl}^{H^+H^-}\right], \tag{4.34}$$

where

$$\begin{aligned} I_{kl}^{H^+H^-} &= \Big\{(C_{-k}^H\, C_{-l}^H + D_{-k}^H\, D_{-l}^H)\,(-2m_\chi^2)\Big[\mathcal{T}_0-\mathcal{Y}_0\Big]+\\ &\quad+(C_{-k}^H\, C_{-l}^H - D_{-k}^H\, D_{-l}^H)(s-2m_\chi^2)\Big[\mathcal{T}_0-\mathcal{Y}_0\Big]\Big\}(s,m_\chi^2,m_{H^\pm}^2,m_{H^\pm}^2,m_{\chi_k^\pm}^2,m_{\chi_l^\pm}^2)\,, \end{aligned} \tag{4.35}$$

$$\begin{aligned} J_{kl}^{H^+H^-} &= C_{-k}^H\, C_{+l}^H\Big[-4\,\mathcal{T}_1-4(m_\chi^2-m_{H^\pm}^2)\mathcal{T}_0-\\ &\quad-2\,\mathcal{Y}_1-2(s-4\,m_\chi^2)\,\mathcal{Y}_0\Big](s,m_\chi^2,m_{H^\pm}^2,m_{H^\pm}^2,m_{\chi_k^\pm}^2,m_{\chi_l^\pm}^2)\,, \end{aligned} \tag{4.36}$$

$$
\begin{aligned}
K_{kl}^{H^+H^-} = \Big\{ & (C^H_{+k}\, C^H_{+l} + D^H_{+k}\, D^H_{+l})\Big[-\mathcal{T}_2 - (s - 2\, m^2_{H^\pm})\, \mathcal{T}_1 - (m^2_\chi - m^2_{H^\pm})^2\, \mathcal{T}_0 + \\
& \qquad + 2m^2_\chi (m^2_\chi - m^2_{H^\pm})\, \mathcal{Y}_0\Big] + \\
& + (C^H_{+k}\, C^H_{+l} - D^H_{+k}\, D^H_{+l})\Big[-2\, m^2_\chi\, \mathcal{T}_1 - \mathcal{Y}_2 + \\
& \qquad + (m^4_\chi - m^4_{H^\pm})\mathcal{Y}_0\Big]\Big\}(s, m^2_\chi, m^2_{H^\pm}, m^2_{H^\pm}, m^2_{\chi^\pm_k}, m^2_{\chi^\pm_l})\,,
\end{aligned} \tag{4.37}
$$

and

$$
C^H_{\pm k} \equiv |C_S^{\chi_k^+ \chi H^-}|^2 \pm |C_P^{\chi_k^+ \chi H^-}|^2\,, \tag{4.38}
$$

$$
D^H_{\pm k} \equiv C_S^{\chi_k^+ \chi H^-} \Big(C_P^{\chi_k^+ \chi H^-}\Big)^* \pm C_P^{\chi_k^+ \chi H^-} \Big(C_S^{\chi_k^+ \chi H^-}\Big)^*\,; \tag{4.39}
$$

Higgs (h, H)-chargino $(\chi^\pm_k)$ interference term:

$$
\begin{aligned}
\widetilde{w}_{H^+H^-}^{(h,H-\chi^\pm)} = 2 \sum_{k=1}^2 \mathrm{Re} & \left[\sum_{r=h,H} \left(\frac{C^{H^+H^-r}\, C_S^{\chi\chi r}}{s - m_r^2 + i\,\Gamma_r\, m_r}\right)^*\right] \times \\
\times\Big\{ & C^H_{+k}\,(-2\, m_\chi) + \Big[C^H_{+k}\, 2\, m_\chi\, (m^2_{H^\pm} - m^2_\chi - m^2_{\chi^\pm_k}) + \\
& + C^H_{-k}\, m_{\chi^\pm_k}\, (s - 4\, m^2_\chi)\Big]\, \mathcal{F}(s, m^2_\chi, m^2_{H^\pm}, m^2_{H^\pm}, m^2_{\chi^\pm_k})\Big\}\,;
\end{aligned} \tag{4.40}
$$

Z-chargino $(\chi^\pm_k)$ interference term:

$$
\begin{aligned}
\widetilde{w}_{H^+H^-}^{(Z-\chi^\pm)} = 2 \sum_{k=1}^2 \mathrm{Re} & \left[\left(\frac{C^{H^+H^-Z}\, C_A^{\chi\chi Z}}{s - m_Z^2 + i\,\Gamma_Z\, m_Z}\right)^*\right] \times D^H_{+k} \\
\times\Big\{ & s - 2\, m^2_{H^\pm} - 2\, m^2_\chi + 2\, m^2_{\chi^\pm_k} \\
& + 2\, \Big[s\, m^2_{\chi^\pm_k} + m^4_{H^\pm} - 2\,(m^2_\chi + m^2_{\chi^\pm_k})\, m^2_{H^\pm} + \\
& + (m^2_\chi - m^2_{\chi^\pm_k})^2\Big]\, \mathcal{F}(s, m^2_\chi, m^2_{H^\pm}, m^2_{H^\pm}, m^2_{\chi^\pm_k})\Big\}.
\end{aligned} \tag{4.41}
$$

4.5 $\chi\chi \to W^+H^-$

This process involves the s-channel CP-even (h, H) and odd (A) Higgs boson exchange as well as the t- and u-channel chargino ($\chi^\pm_k$, $k = 1, 2$) exchange

$$
\widetilde{w}_{WH} = \widetilde{w}_{WH}^{(h,H)} + \widetilde{w}_{WH}^{(A)} + \widetilde{w}_{WH}^{(\chi^\pm)} + \widetilde{w}_{WH}^{(h,H-\chi^\pm)} + \widetilde{w}_{WH}^{(A-\chi^\pm)}\ : \tag{4.42}
$$

CP-even Higgs-boson (h, H) exchange:

$$
\begin{aligned}
\widetilde{w}_{WH}^{(h,H)} = \frac{1}{2} & \left|\sum_{r=h,H} \frac{C^{W^-H^+r}\, C_S^{\chi\chi i}}{s - m_r^2 + i\,\Gamma_r\, m_r}\right|^2 (s - 4\, m^2_\chi) \times \\
& \times \frac{s^2 - 2\,(m^2_{H^\pm} + m^2_W)\, s + (m^2_{H^\pm} - m^2_W)^2}{m^2_W}\,;
\end{aligned} \tag{4.43}
$$

CP-odd Higgs-boson (A) exchange:

$$\widetilde{w}_{WH}^{(A)} = \frac{1}{2}\left|\frac{C^{W^-H^+A}\,C_P^{\chi\chi A}}{s-m_A^2+i\,\Gamma_A\,m_A}\right|^2 s \\ \times\frac{s^2-2\,(m_{H^\pm}^2+m_W^2)\,s+(m_{H^\pm}^2-m_W^2)^2}{m_W^2}\,; \tag{4.44}$$

Chargino ($\chi_k^\pm$) exchange:

$$\widetilde{w}_{WH}^{(\chi^\pm)} = \frac{1}{m_W^2}\sum_{k,l=1}^{2}\left[m_{\chi_k^\pm}\,m_{\chi_l^\pm}I_{kl}^{WH}+m_\chi m_{\chi_k^\pm}J_{kl}^{WH}+K_{kl}^{WH}\right], \tag{4.45}$$

where

$$\begin{aligned}
I_{kl}^{WH} = &\Big\{(C_{+k}^{HW*}C_{+l}^{HW}+D_{+k}^{HW*}D_{+l}^{HW})\times\\
&\times\left[-\mathcal{T}_2-(s-m_W^2-m_{H^\pm}^2-2\,m_\chi^2)\,\mathcal{T}_1+G_{WH}^{T(1)}\,\mathcal{T}_0+6\,m_W^2m_\chi^2\,\mathcal{Y}_0\right]+\\
&+(C_{+k}^{HW*}C_{+l}^{HW}-D_{+k}^{HW*}D_{+l}^{HW})\times\\
&\times\left[6\,m_W^2m_\chi^2\,\mathcal{T}_0-\mathcal{Y}_2+G_{WH}^{Y(1)}\,\mathcal{Y}_0\right]\Big\}(s,m_\chi^2,m_{H^\pm}^2,m_W^2,m_{\chi_k^\pm}^2,m_{\chi_l^\pm}^2)\,, &(4.46)\\
J_{kl}^{WH} = &\Big\{\mathrm{Re}(C_{+k}^{HW*}C_{-l}^{HW}+D_{+k}^{HW*}D_{-l}^{HW})\times\\
&\times\Big[-2\mathcal{T}_2+2(2m_\chi^2-m_W^2)\mathcal{T}_1-2(m_\chi^2-m_W^2)(m_\chi^2+2m_W^2)\mathcal{T}_0-\\
&-3m_W^2\mathcal{Y}_1-G_{WH}^{Y(2)}\mathcal{Y}_0\Big]+\mathrm{Re}(C_{+k}^{HW*}C_{-l}^{HW}-D_{+k}^{HW*}D_{-l}^{HW})\times\\
&\times\Big[6m_W^2\mathcal{T}_1+6m_W^2(m_\chi^2-m_{H^\pm}^2)\mathcal{T}_0-2\mathcal{Y}_2-(s-m_{H^\pm}^2-2m_W^2)\mathcal{Y}_1-\\
&-G_{WH}^{Y(3)}\mathcal{Y}_0\Big]\Big\}(s,m_\chi^2,m_{H^\pm}^2,m_W^2,m_{\chi_k^\pm}^2,m_{\chi_l^\pm}^2)\,, &(4.47)\\
K_{kl}^{WH} = &\Big\{(C_{-k}^{HW*}C_{-l}^{HW}+D_{-k}^{HW*}D_{-l}^{HW})\times\\
&\times\left[(s-m_\chi^2-2\,m_W^2)\mathcal{T}_2-G_{WH}^{T(2)}\mathcal{T}_1-G_{WH}^{T(3)}\mathcal{T}_0+(s-2\,m_W^2)\mathcal{Y}_2+G_{WH}^{Y(4)}\mathcal{Y}_0\right]+\\
&+(C_{-k}^{HW*}C_{-l}^{HW}-D_{-k}^{HW*}D_{-l}^{HW})\times\\
&\times\left[6m_W^2m_\chi^2\mathcal{T}_1-m_\chi^2\mathcal{Y}_2+G_{WH}^{Y(5)}\mathcal{Y}_0\right]\Big\}(s,m_\chi^2,m_{H^\pm}^2,m_W^2,m_{\chi_k^\pm}^2,m_{\chi_l^\pm}^2)\,, &(4.48)
\end{aligned}$$

and

$$\begin{aligned}
G_{WH}^{T(1)} &= s(m_\chi^2+2m_W^2)-m_\chi^4-m_\chi^2(m_{H^\pm}^2+3m_W^2)-m_W^2m_{H^\pm}^2\,,\\
G_{WH}^{T(2)} &= s(m_\chi^2+2m_W^2)-2m_\chi^4+m_\chi^2(m_{H^\pm}^2-m_W^2)-2m_W^2(m_{H^\pm}^2+m_W^2)\,,\\
G_{WH}^{T(3)} &= (m_\chi^2-m_{H^\pm}^2)(m_\chi^2-m_W^2)(m_\chi^2+2m_W^2)\,,\\
G_{WH}^{Y(1)} &= G_{WH}^{T(1)}\,,
\end{aligned}$$

$$
\begin{aligned}
G_{WH}^{Y(2)} &= 3\, s\, m_W^2 - 12\, m_W^2 m_\chi^2 + 3\, m_W^2 m_{H^\pm}^2 - 3\, m_W^4\,, \\
G_{WH}^{Y(3)} &= s^2 - s(2m_\chi^2 + 2m_{H^\pm}^2 + 3m_W^2) + 2m_\chi^4 + (2m_\chi^2 + m_{H^\pm}^2)(m_{H^\pm}^2 + 3m_W^2) - 2m_W^4\,, \\
G_{WH}^{Y(4)} &= s\,(m_\chi^2 - 2m_W^2)(m_\chi^2 - m_{H^\pm}^2) + 4m_W^2 m_\chi^4 + \\
&\quad + m_\chi^2 (m_{H^\pm}^4 - 5m_W^2 m_{H^\pm}^2 + 2m_W^4) - 2m_W^4 m_{H^\pm}^2\,, \\
G_{WH}^{Y(5)} &= s\, m_\chi^2 (m_\chi^2 - m_W^2) - m_\chi^6 - m_\chi^4 (m_{H^\pm}^2 + 3m_W^2) + m_\chi^2 m_W^2 (2m_{H^\pm}^2 + 3m_W^2)\,,
\end{aligned}
$$

and

$$
C_{\pm k}^{HW} \equiv C_S^{\chi_k^+ \chi H^-} \left(C_V^{\chi_k^+ \chi W^-} \right)^* \pm\, C_P^{\chi_k^+ \chi H^-} \left(C_A^{\chi_k^+ \chi W^-} \right)^*, \tag{4.49}
$$

$$
D_{\pm k}^{HW} \equiv C_S^{\chi_k^+ \chi H^-} \left(C_A^{\chi_k^+ \chi W^-} \right)^* \pm\, C_P^{\chi_k^+ \chi H^-} \left(C_V^{\chi_k^+ \chi W^-} \right)^*; \tag{4.50}
$$

Higgs (h, H)-chargino $(\chi_k^\pm)$ interference term:

$$
\begin{aligned}
\widetilde{w}_{WH}^{(h,H-\chi^\pm)} &= -2\,\mathrm{Re} \sum_{k=1}^{2} \sum_{r=h,H} \left(\frac{C^{W^- H^+ r}\, C_S^{\chi\chi r}}{s - m_r^2 + i\,\Gamma_r m_r} \right)^* \frac{1}{m_W^2} \times \\
&\quad \times \left[C_{+k}^{HW*} H_k^{(+)WH} + C_{-k}^{HW*} H_k^{(-)WH} \right],
\end{aligned} \tag{4.51}
$$

where

$$
\begin{aligned}
H_k^{(+)WH} &= 2\, m_\chi\, m_{\chi_k^\pm}\, (s + m_W^2 - m_{H^\pm}^2) + \\
&\quad + m_\chi\, m_{\chi_k^\pm} \Big[s^2 + 2s(m_{\chi_k^\pm} - m_\chi^2 - m_{H^\pm}^2) + m_{H^\pm}^4 - m_W^4 + \\
&\qquad + 2\,(m_{\chi_k^\pm}^2 - m_\chi^2)\,(m_W^2 - m_{H^\pm}^2) \Big] \mathcal{F}(s, m_\chi^2, m_{H^\pm}^2, m_W^2, m_{\chi_k^\pm}^2)\,,
\end{aligned} \tag{4.52}
$$

$$
\begin{aligned}
H_k^{(-)WH} &= -s^2 + s(m_W^2 + m_{H^\pm}^2 + 2\, m_\chi^2) + 2\, m_\chi^2 (m_W^2 - m_{H^\pm}^2) + \\
&\quad + \Big\{ -s^2\, m_{\chi_k^\pm}^2 + s \Big[2m_\chi^2 (m_{\chi_k^\pm}^2 - m_\chi^2) + m_\chi^2 (m_{H^\pm}^2 + 3\, m_W^2) + \\
&\qquad + m_{\chi_k^\pm}^2 (m_{H^\pm}^2 + m_W^2) - 2\, m_W^2\, m_{H^\pm}^2 \Big] + \\
&\qquad + m_\chi^2 (m_W^2 - m_{H^\pm}^2)(2m_{\chi_k^\pm}^2 - 2m_\chi^2 + m_{H^\pm}^2 - 3m_W^2) \Big\} \times \\
&\quad \times \mathcal{F}(s, m_\chi^2, m_{H^\pm}^2, m_W^2, m_{\chi_k^\pm}^2)\,;
\end{aligned} \tag{4.53}
$$

Higgs (A)-chargino $(\chi_k^\pm)$ interference term:

$$
\begin{aligned}
\widetilde{w}_{WH}^{(A-\chi^\pm)} &= -2\,\mathrm{Re} \sum_{k=1}^{2} \left(\frac{C^{W^- H^+ A}\, C_P^{\chi\chi A}}{s - m_A^2 + i\,\Gamma_A\, m_A} \right)^* \frac{1}{m_W^2} \times \\
&\quad \times \left[D_{+k}^{HW*} A_k^{(+)WH} + D_{-k}^{HW*} A_k^{(-)WH} \right],
\end{aligned} \tag{4.54}
$$

where

$$
A_k^{(+)WH} = m_\chi m_{\chi_k^\pm} \left[s^2 - 2s(m_{H^\pm}^2 + m_W^2) + (m_{H^\pm}^2 - m_W^2)^2 \right] \mathcal{F}(s, m_\chi^2, m_{H^\pm}^2, m_W^2, m_{\chi_k^\pm}^2)\,, \tag{4.55}
$$

$$
\begin{aligned}
A_k^{(-)WH} &= s\,(s - m_W^2 - m_{H^\pm}^2) + \Big\{ s^2\, m_{\chi_k^\pm}^2 - s\,[(m_\chi^2 + m_{\chi_k^\pm}^2)\,(m_{H^\pm}^2 + m_W^2) - \\
&\qquad - 2\, m_W^2\, m_{H^\pm}^2] + m_\chi^2\,(m_{H^\pm}^2 - m_W^2)^2 \Big\} \times \\
&\quad \times \mathcal{F}(s, m_\chi^2, m_{H^\pm}^2, m_W^2, m_{\chi_k^\pm}^2)\,.
\end{aligned} \tag{4.56}
$$

Note that $\widetilde{w}_{WH}$ in eq. (4.42) does not include the contribution for W^-H^+ final state. The contributions to the W^-H^+ and W^+H^- final states are obviously identical and so the total contribution for $W^\pm H^\mp$ is twice that of W^+H^-.

4.6 $\chi\chi \to Zh$

This process proceeds via the s-channel Z and CP-odd Higgs boson (A) exchange as well as the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{w}_{Zh} = \widetilde{w}_{Zh}^{(A)} + \widetilde{w}_{Zh}^{(Z)} + \widetilde{w}_{Zh}^{(\chi^0)} + \widetilde{w}_{Zh}^{(A-Z)} + \widetilde{w}_{Zh}^{(A-\chi^0)} + \widetilde{w}_{Zh}^{(Z-\chi^0)} : \tag{4.57}$$

CP-odd Higgs-boson (A) exchange:

$$\widetilde{w}_{Zh}^{(A)} = \frac{1}{2} \left| \frac{C^{ZhA}\, C_P^{\chi\chi A}}{s - m_A^2 + i\,\Gamma_A\, m_A} \right|^2 s\, \frac{s^2 - 2\,(m_h^2 + m_Z^2)\, s + (m_h^2 - m_Z^2)^2}{m_Z^2} ; \tag{4.58}$$

Z-boson exchange:

$$\begin{aligned}
\widetilde{w}_{Zh}^{(Z)} = {} & \frac{1}{12} \left| \frac{C^{ZZh}\, C_A^{\chi\chi Z}}{s - m_Z^2 + i\,\Gamma_Z\, m_Z} \right|^2 \frac{1}{m_Z^6\, s} \times \\
& \times \Big\{ m_\chi^2 \Big[6s^4 - 12(m_h^2 + 2m_Z^2)s^3 + (32m_Z^4 + 12m_Z^2 m_h^2 + 6m_h^4)\, s^2 - \\
& \quad - (64m_Z^4 - 20\, m_Z^2 m_h^2 + 12m_h^4) s m_Z^2 + 2(m_h^2 - m_Z^2)^2 m_Z^4 \Big] + \\
& \quad + m_Z^4 [s^3 + (10m_Z^2 - 2m_h^2) s^2 + (m_h^2 - m_Z^2)\, s] \Big\} ;
\end{aligned} \tag{4.59}$$

Neutralino (χ_i^0) exchange:

$$\widetilde{w}_{Zh}^{(\chi^0)} = \frac{1}{m_Z^2} \sum_{i,j=1}^{4} C_S^{\chi_i^0 \chi h} C_S^{\chi_j^0 \chi h *} \left[m_{\chi_i^0}\, m_{\chi_j^0} I_{ij}^{Zh} + m_\chi m_{\chi_i^0} J_{ij}^{Zh} + K_{ij}^{Zh} \right] , \tag{4.60}$$

where

$$\begin{aligned}
I_{ij}^{Zh} = {} & \Big\{ (C_{Vi}^\chi C_{Vj}^{\chi *} + C_{Ai}^\chi C_{Aj}^{\chi *}) \times \\
& \times \Big[-\mathcal{T}_2 - (s - m_Z^2 - m_h^2 - 2m_\chi^2)\mathcal{T}_1 + G_{Zh}^{T(1)} \mathcal{T}_0 + 6m_Z^2 m_\chi^2 \mathcal{Y}_0 \Big] + \\
& + (C_{Vi}^\chi C_{Vj}^{\chi *} - C_{Ai}^\chi C_{Aj}^{\chi *}) \Big[6m_Z^2 m_\chi^2 \mathcal{T}_0 - \mathcal{Y}_2 + \\
& \qquad + G_{Zh}^{Y(1)} \mathcal{Y}_0 \Big] \Big\} (s, m_\chi^2, m_h^2, m_Z^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2) ,
\end{aligned} \tag{4.61}$$

$$\begin{aligned}
J_{ij}^{Zh} = {} & \Big\{ (C_{Vi}^\chi C_{Vj}^{\chi *} + C_{Ai}^\chi C_{Aj}^{\chi *}) \times \\
& \times \Big[-2\mathcal{T}_2 + 2(2m_\chi^2 - m_Z^2)\mathcal{T}_1 - 2(m_\chi^2 - m_Z^2)(m_\chi^2 + 2m_Z^2)\mathcal{T}_0 - 3m_Z^2 \mathcal{Y}_1 - G_{Zh}^{Y(2)} \mathcal{Y}_0 \Big] + \\
& + (C_{Vi}^\chi C_{Vj}^{\chi *} - C_{Ai}^\chi C_{Aj}^{\chi *}) \times \\
& \times \Big[6m_Z^2 \mathcal{T}_1 + 6m_Z^2 (m_\chi^2 - m_h^2)\mathcal{T}_0 - -2\mathcal{Y}_2 - (s - m_h^2 - 2m_Z^2)\mathcal{Y}_1 -
\end{aligned}$$

$$- G_{Zh}^{Y(3)}\mathcal{Y}_0\Big]\Big\}(s,m_\chi^2,m_h^2,m_Z^2,m_{\chi_i^0}^2,m_{\chi_j^0}^2)\,, \tag{4.62}$$

$$\begin{aligned}
K_{ij}^{Zh} = &\Big\{(C_{Vi}^{\chi}C_{Vj}^{\chi *}+C_{Ai}^{\chi}C_{Aj}^{\chi *})\times \\
&\times\Big[(s-m_\chi^2-2\,m_Z^2)\mathcal{T}_2-G_{Zh}^{T(2)}\mathcal{T}_1-G_{Zh}^{T(3)}\mathcal{T}_0+(s-2m_Z^2)\mathcal{Y}_2+G_{Zh}^{Y(4)}\mathcal{Y}_0\Big]+ \\
&+(C_{Vi}^{\chi}C_{Vj}^{\chi *}-C_{Ai}^{\chi}C_{Aj}^{\chi *})\Big[6m_Z^2m_\chi^2\mathcal{T}_1-m_\chi^2\mathcal{Y}_2+ \\
&\qquad +G_{ZH}^{Y(5)}\mathcal{Y}_0\Big]\Big\}(s,m_\chi^2,m_h^2,m_Z^2,m_{\chi_i^0}^2,m_{\chi_j^0}^2)\,,
\end{aligned} \tag{4.63}$$

and

$$\begin{aligned}
G_{Zh}^{T(1)} &= s(m_\chi^2+2m_Z^2)-m_\chi^4-m_\chi^2(m_h^2+3m_Z^2)-m_Z^2m_h^2\,,\\
G_{Zh}^{T(2)} &= s(m_\chi^2+2m_Z^2)-2m_\chi^4+m_\chi^2(m_h^2-m_Z^2)-2m_Z^2(m_h^2+m_Z^2)\,,\\
G_{Zh}^{T(3)} &= (m_\chi^2-m_h^2)\,(m_\chi^2-m_Z^2)(m_\chi^2+2\,m_Z^2)\,,\\
G_{Zh}^{Y(1)} &= G_{Zh}^{T(1)}\,,\\
G_{Zh}^{Y(2)} &= 3sm_Z^2-12m_Z^2m_\chi^2+3m_Z^2m_h^2-3m_Z^4\,,\\
G_{Zh}^{Y(3)} &= s^2-s(2m_\chi^2+2m_h^2+3m_Z^2)+2m_\chi^4+(2m_\chi^2+m_h^2)(m_h^2+3m_Z^2)-2m_Z^4\,,\\
G_{Zh}^{Y(4)} &= s(m_\chi^2-2m_Z^2)(m_\chi^2-m_h^2)+4m_Z^2m_\chi^4+\\
&\quad +m_\chi^2(m_h^4-5m_Z^2m_h^2+2m_Z^4)-2m_Z^4m_h^2\,,\\
G_{Zh}^{Y(5)} &= sm_\chi^2(m_\chi^2-m_Z^2)-m_\chi^6-m_\chi^4(m_h^2+3m_Z^2)+m_\chi^2m_Z^2(2m_h^2+3m_Z^2)\,,
\end{aligned}$$

and

$$C_{Vi}^{\chi}\equiv C_V^{\chi_i^0\chi Z}\,, \tag{4.64}$$

$$C_{Ai}^{\chi}\equiv C_A^{\chi_i^0\chi Z}\,; \tag{4.65}$$

Higgs (A)-Z interference term:

$$\begin{aligned}
\widetilde{w}_{Zh}^{(A-Z)} &= \mathrm{Re}\left[\left(\frac{C^{ZhA}\,C_P^{\chi\chi A}}{s-m_A^2+i\Gamma_A\,m_A}\right)^*\frac{C^{ZZh}C_A^{\chi\chi Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right]\times\\
&\quad\times\frac{m_\chi\,(m_Z^2-s)}{m_Z^4}\,[s^2-2\,(m_h^2+m_Z^2)\,s+(m_h^2-m_Z^2)^2]\,;
\end{aligned} \tag{4.66}$$

Higgs (A)-neutralino (χ_i^0) interference term:

$$\begin{aligned}
\widetilde{w}_{Zh}^{(A-\chi^0)} &= 2\sum_{i=1}^{4}\mathrm{Re}\left[\left(\frac{C^{ZhA}\,C_P^{\chi\chi A}}{s-m_A^2+i\,\Gamma_A\,m_A}\right)^* C_S^{\chi_i^0\chi h}C_A^{\chi_i^0\chi Z}\right]\times\\
&\quad\times\Big[-s^2+(m_h^2+m_Z^2)\,s-\\
&\qquad -\Big\{s^2m_{\chi_i^0}(m_\chi+m_{\chi_i^0})+s\Big[2m_h^2m_Z^2-(m_h^2+m_Z^2)(m_\chi+m_{\chi_i^0})^2\Big]+\\
&\qquad\quad +m_\chi(m_\chi+m_{\chi_i^0})(m_h^2-m_Z^2)^2\Big\}\mathcal{F}(s,m_\chi^2,m_h^2,m_Z^2,m_{\chi_i^0}^2)\Big]\,;
\end{aligned} \tag{4.67}$$

Z-neutralino (χ_i^0) interference term:

$$
\begin{aligned}
\widetilde{w}_{Zh}^{(Z-\chi^0)} &= -\sum_{i=1}^{4} \mathrm{Re}\left[\left(\frac{C^{ZZh}\, C_A^{\chi\chi Z}}{s-m_Z^2+i\,\Gamma_Z\, m_Z}\right)^{*} C_S^{\chi_i^0\chi h}\, C_A^{\chi_i^0\chi Z}\right]\frac{1}{m_Z^4}\times \\
&\quad\times\Big\{m_\chi\Big[-2\,s^2+s(2m_h^2+3\,m_Z^2)+m_Z^2\,(2\,m_\chi^2-9m_Z^2-m_h^2-2\,m_{\chi_i^0}^2)\Big]+ \\
&\qquad+m_{\chi_i^0}\, m_Z^2\,\Big[-s+(2m_\chi^2+m_Z^2+m_h^2-2\,m_{\chi_i^0}^2)\Big]- \\
&\qquad-2\Big[m_\chi\Big\{s^2\, m_{\chi_i^0}^2+s\Big[-(m_\chi^2+m_{\chi_i^0}^2)(m_Z^2+m_h^2)+2m_Z^2 m_h^2\Big]- \\
&\qquad\qquad-2m_Z^6+5m_\chi^2 m_Z^4+4m_{\chi_i^0}^2 m_Z^4+m_Z^2(m_\chi^2-m_{\chi_i^0}^2)^2- \\
&\qquad\qquad-m_h^2 m_Z^2(3m_Z^2+2m_\chi^2)+m_\chi^2 m_h^4\Big\}+ \\
&\qquad\quad+m_{\chi_i^0}\Big[s^2 m_\chi^2+s(-3m_\chi^2 m_Z^2+m_Z^2 m_{\chi_i^0}^2-2m_\chi^2 m_h^2-2m_Z^4)+ \\
&\qquad\qquad+10m_\chi^2 m_Z^4-m_{\chi_i^0}^2 m_Z^4+m_Z^2(m_\chi^2-m_{\chi_i^0}^2)^2+ \\
&\qquad\qquad+m_h^2 m_Z^2(m_Z^2-m_\chi^2-m_{\chi_i^0}^2)+m_\chi^2 m_h^4\Big]\Big]\mathcal{F}(s,m_\chi^2,m_h^2,m_Z^2,m_{\chi_i^0}^2)\Big\}.
\end{aligned}
\tag{4.68}
$$

The expressions for ZH final state are found by replacing h with H in the above.

4.7 $\chi\chi \to ZA$

This process involves the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i=1,\ldots,4$) exchange

$$
\widetilde{w}_{ZA} = \widetilde{w}_{ZA}^{(h,H)} + \widetilde{w}_{ZA}^{(\chi^0)} + \widetilde{w}_{ZA}^{(h,H-\chi^0)} \; : \tag{4.69}
$$

CP-even Higgs-boson (h,H) exchange:

$$
\widetilde{w}_{ZA}^{(h,H)} = \frac{1}{2}\left|\sum_{r=h,H}\frac{C^{ZAr}C_S^{\chi\chi r}}{s-m_r^2+i\Gamma_r m_r}\right|^2 (s-4m_\chi^2)\frac{s^2-2(m_A^2+m_Z^2)s+(m_A^2-m_Z^2)^2}{m_Z^2}\; ; \tag{4.70}
$$

Neutralino (χ_i^0) exchange:

$$
\widetilde{w}_{ZA}^{(\chi^0)} = \frac{1}{m_Z^2}\sum_{i,j=1}^{4} C_P^{\chi_i^0\chi A} C_P^{\chi_j^0\chi A*}\Big[m_{\chi_i^0}\, m_{\chi_j^0} I_{ij}^{ZA} + m_\chi m_{\chi_i^0} J_{ij}^{ZA} + K_{ij}^{ZA}\Big], \tag{4.71}
$$

where

$$
\begin{aligned}
I_{ij}^{ZA} &= \Big\{(C_{Vi}^{\chi} C_{Vj}^{\chi*} + C_{Ai}^{\chi} C_{Aj}^{\chi*})\times \\
&\quad\times\Big[-\mathcal{T}_2-(s-m_Z^2-m_A^2-2m_\chi^2)\,\mathcal{T}_1+G_{ZA}^{T(1)}\mathcal{T}_0+6m_Z^2 m_\chi^2\mathcal{Y}_0\Big]+ \\
&\quad+(C_{Vi}^{\chi} C_{Vj}^{\chi*} - C_{Ai}^{\chi} C_{Aj}^{\chi*})\Big[-6m_Z^2 m_\chi^2\mathcal{T}_0+\mathcal{Y}_2-G_{ZA}^{Y(1)}\mathcal{Y}_0\Big]\Big\}(s,m_\chi^2,m_A^2,m_Z^2,m_{\chi_i^0}^2,m_{\chi_j^0}^2),
\end{aligned}
\tag{4.72}
$$

$$
\begin{aligned}
J_{ij}^{ZA} = \Big\{ & (C_{Vi}^{\chi} C_{Vj}^{\chi *} + C_{Ai}^{\chi} C_{Aj}^{\chi *}) \Big[2\mathcal{T}_2 - 2(2m_\chi^2 - m_Z^2)\mathcal{T}_1 + \\
& \qquad + 2(m_\chi^2 - m_Z^2)(m_\chi^2 + 2\, m_Z^2)\mathcal{T}_0 + 3m_Z^2 \mathcal{Y}_1 + G_{ZA}^{Y(2)} \mathcal{Y}_0 \Big] + \\
& + (C_{Vi}^{\chi} C_{Vj}^{\chi *} - C_{Ai}^{\chi} C_{Aj}^{\chi *}) \Big[6m_Z^2 \mathcal{T}_1 + 6m_Z^2 (m_\chi^2 - m_A^2)\mathcal{T}_0 - \\
& \qquad - 2\mathcal{Y}_2 - (s - m_A^2 - 2m_Z^2)\mathcal{Y}_1 - \\
& \qquad - G_{ZA}^{Y(3)} \mathcal{Y}_0 \Big] \Big\} (s, m_\chi^2, m_A^2, m_Z^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \qquad (4.73)
\end{aligned}
$$

$$
\begin{aligned}
K_{ij}^{ZA} = \Big\{ & (C_{Vi}^{\chi} C_{Vj}^{\chi *} + C_{Ai}^{\chi} C_{Aj}^{\chi *}) \Big[(s - m_\chi^2 - 2m_Z^2)\mathcal{T}_2 - G_{ZA}^{T(2)} \mathcal{T}_1 - \\
& \qquad - G_{ZA}^{T(3)} \mathcal{T}_0 + (s - 2m_Z^2)\mathcal{Y}_2 + G_{ZA}^{Y(4)} \mathcal{Y}_0 \Big] + \\
& + (C_{Vi}^{\chi} C_{Vj}^{\chi *} - C_{Ai}^{\chi} C_{Aj}^{\chi *}) \Big[-6\, m_Z^2 m_\chi^2\, \mathcal{T}_1 + m_\chi^2\, \mathcal{Y}_2 - \\
& \qquad - G_{ZA}^{Y(5)} \mathcal{Y}_0 \Big] \Big\} (s, m_\chi^2, m_A^2, m_Z^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2), \qquad (4.74)
\end{aligned}
$$

and

$$
\begin{aligned}
G_{ZA}^{T(1)} &= s(m_\chi^2 + 2m_Z^2) - m_\chi^4 - m_\chi^2 (m_A^2 + 3m_Z^2) - m_Z^2 m_A^2\,, \\
G_{ZA}^{T(2)} &= s(m_\chi^2 + 2m_Z^2) - 2m_\chi^4 + m_\chi^2 (m_A^2 - m_Z^2) - 2m_Z^2 (m_A^2 + m_Z^2)\,, \\
G_{ZA}^{T(3)} &= (m_\chi^2 - m_A^2)\,(m_\chi^2 - m_Z^2)\,(m_\chi^2 + 2\, m_Z^2)\,, \\
G_{ZA}^{Y(1)} &= G_{ZA}^{T(1)}\,, \\
G_{ZA}^{Y(2)} &= 3\, s\, m_Z^2 - 12\, m_Z^2 m_\chi^2 + 3\, m_Z^2 m_A^2 - 3\, m_Z^4\,, \\
G_{ZA}^{Y(3)} &= s^2 - s(2m_\chi^2 + 2m_A^2 + 3m_Z^2) + 2m_\chi^4 + (2m_\chi^2 + m_A^2)(m_A^2 + 3m_Z^2) - 2m_Z^4\,, \\
G_{ZA}^{Y(4)} &= s\,(m_\chi^2 - 2m_Z^2)(m_\chi^2 - m_A^2) + 4m_Z^2 m_\chi^4 + \\
&\quad + m_\chi^2 (m_A^4 - 5m_Z^2 m_A^2 + 2m_Z^4) - 2m_Z^4 m_A^2\,, \\
G_{ZA}^{Y(5)} &= s\, m_\chi^2 (m_\chi^2 - m_Z^2) - m_\chi^6 - m_\chi^4 (m_A^2 + 3m_Z^2) + m_\chi^2 m_Z^2 (2m_A^2 + 3m_Z^2)\,.
\end{aligned}
$$

$G_{ZA}^{T(1-3)}$ and $G_{ZA}^{Y(1-5)}$ are obtained from $G_{Zh}^{T(1-3)}$ and $G_{Zh}^{Y(1-5)}$ by replacing m_h with m_A. C_{Vi}^{χ} and C_{Ai}^{χ} have already been defined for the Zh final state in eqs. (4.64) and (4.65).

Higgs (h, H)-neutralino (χ_i^0) interference term:

$$
\begin{aligned}
\widetilde{w}_{ZA}^{(h,H-\chi^0)} = & -2 \sum_{i=1}^{4} \mathrm{Re} \left[\sum_{r=h,H} \left(\frac{C^{ZAr} C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r} \right)^* C_A^{\chi_i^0 \chi Z} C_P^{\chi_i^0 \chi A} \right] \times \\
& \times \Big[-s^2 + (m_A^2 + m_Z^2)s + 2m_\chi (m_{\chi_i^0} - m_\chi)(-s - m_Z^2 + m_A^2) + \\
& \quad + \Big\{ -(m_{\chi_i^0} + m_\chi)(s - m_A^2) \Big[m_{\chi_i^0}\, s + 2m_\chi (m_{\chi_i^0} - m_\chi)^2 - m_\chi m_A^2 \Big] + \\
& \qquad + m_Z^2 m_\chi \Big[-2(m_{\chi_i^0} - m_\chi)^2 (m_{\chi_i^0} + m_\chi) + 4m_\chi m_A^2
\end{aligned}
$$

$$+ m_Z^2(m_{\chi_i^0} - 3m_\chi) + 3sm_\chi\Big] + sm_Z^2(m_{\chi_i^0}^2 - 2m_A^2)\Big\} \times$$

$$\times\mathcal{F}(s, m_\chi^2, m_A^2, m_Z^2, m_{\chi_i^0}^2)\Big] . \tag{4.75}$$

4.8 $\chi\chi \to WW$

This process involves the s-channel CP-even Higgs boson (h and H) and Z exchange, and the t- and u-channel chargino ($\chi_k^\pm$, $k = 1, 2$) exchange

$$\widetilde{w}_{WW} = \widetilde{w}_{WW}^{(h,H)} + \widetilde{w}_{WW}^{(Z)} + \widetilde{w}_{WW}^{(\chi^\pm)} + \widetilde{w}_{WW}^{(h,H-\chi^\pm)} + \widetilde{w}_{WW}^{(Z-\chi^\pm)} \; : \tag{4.76}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{w}_{WW}^{(h,H)} = \left|\sum_{r=h,H} \frac{C^{WWr} C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r}\right|^2 (s - 4m_\chi^2)\frac{s^2 - 4m_W^2 s + 12m_W^4}{8m_W^4} \, ; \tag{4.77}$$

Z-boson exchange:

$$\widetilde{w}_{WW}^{(Z)} = \left|\frac{C^{WWZ} C_A^{\chi\chi Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right|^2 (s - 4m_\chi^2)\frac{s^3 + 16m_W^2 s^2 - 68m_W^4 s - 48m_W^6}{12m_W^4} \, ; \tag{4.78}$$

Chargino $(\chi_k^\pm)$ exchange:

$$\widetilde{w}_{WW}^{(\chi^\pm)} = \frac{1}{m_W^4}\sum_{k,l=1}^{2}\left[m_{\chi_k^\pm} m_{\chi_l^\pm} I_{kl}^{WW} + m_\chi m_{\chi_k^\pm} J_{kl}^{WW} + K_{kl}^{WW}\right] , \tag{4.79}$$

where

$$\begin{aligned}
I_{kl}^{WW} = & \Big\{(C_{-k}^{W*} C_{-l}^{W} + D_{-k}^{W*} D_{-l}^{W}) \times \\
& \times \Big[(s - 4m_W^2)\mathcal{T}_2 - G_{WW}^{T(1)}\mathcal{T}_1 + G_{WW}^{T(2)}\mathcal{T}_0 + (s - 4m_W^2)\mathcal{Y}_2 - G_{WW}^{Y(1)}\mathcal{Y}_0\Big] + \\
& + (C_{-k}^{W*} C_{-l}^{W} - D_{-k}^{W*} D_{-l}^{W})\Big[-18\, m_W^4 m_\chi^2 \mathcal{T}_0 - \\
& - m_\chi^2(-s^2 + 4m_W^2 s + 6m_W^4)\mathcal{Y}_0\Big]\Big\}(s, m_\chi^2, m_W^2, m_W^2, m_{\chi_k^\pm}^2, m_{\chi_l^\pm}^2) ,
\end{aligned} \tag{4.80}$$

$$\begin{aligned}
J_{kl}^{WW} = & \Big\{C_{-k}^{W*} C_{+l}^{W}\Big[12m_W^2\mathcal{T}_2 + 12m_W^2(m_W^2 - 2m_\chi^2)\mathcal{T}_1 + \\
& + 12m_W^2\,(m_\chi^4 + m_W^2 m_\chi^2 - 2m_W^4)\mathcal{T}_0 - 4\,(s - m_W^2)\mathcal{Y}_2 + \\
& + 2m_W^2(2s - 3m_W^2)\mathcal{Y}_1 + G_{WW}^{Y(2)}\mathcal{Y}_0\Big]\Big\}(s, m_\chi^2, m_W^2, m_W^2, m_{\chi_k^\pm}^2, m_{\chi_l^\pm}^2) ,
\end{aligned} \tag{4.81}$$

$$\begin{aligned}
K_{kl}^{WW} = & \Big\{(C_{+k}^{W*} C_{+l}^{W} + D_{+k}^{W*} D_{+l}^{W})\Big[-\mathcal{T}_4 - (s - 2m_W^2 - 4m_\chi^2)\mathcal{T}_3 - G_{WW}^{T(3)}\mathcal{T}_2 - \\
& - G_{WW}^{T(4)}\mathcal{T}_1 - (m_W^2 - m_\chi^2)^2(2m_W^2 + m_\chi^2)\mathcal{T}_0 + \\
& + m_\chi^2(s - 4m_W^2)\mathcal{Y}_2 - G_{WW}^{Y(3)}\mathcal{Y}_0\Big] +
\end{aligned}$$

$$+ (C^{W*}_{+k}C^W_{+l} - D^{W*}_{+k}D^W_{+l})\Big[-18m_W^4 m_\chi^2 \mathcal{T}_1 + \mathcal{Y}_4 + \\ + (2m_\chi^4 - 4sm_W^2 + m_W^4)\mathcal{Y}_2 - \\ - G^{Y(4)}_{WW}\mathcal{Y}_0\Big]\Big\}(s, m_\chi^2, m_W^2, m_W^2, m^2_{\chi^\pm_k}, m^2_{\chi^\pm_l}), \quad (4.82)$$

and

$$\begin{aligned}
G^{T(1)}_{WW} &= 2[s(m_\chi^2 + 2m_W^2) - 4m_W^2(m_\chi^2 + m_W^2)],\\
G^{T(2)}_{WW} &= s(m_\chi^2 + 2m_W^2)^2 + m_W^2(-4m_\chi^4 - 10m_\chi^2 m_W^2 - 4m_W^4),\\
G^{T(3)}_{WW} &= -2s(m_\chi^2 + 2m_W^2) + 6m_\chi^4 + 6m_\chi^2 m_W^2 + 5m_W^4,\\
G^{T(4)}_{WW} &= s(m_\chi^2 + 2m_W^2)^2 - 8m_W^6 - 6m_\chi^4 m_W^2 - 4m_\chi^6,\\
G^{Y(1)}_{WW} &= -sm_\chi^2(m_\chi^2 + 2m_W^2) + 6m_\chi^2 m_W^4 + 4m_W^2(m_\chi^2 - m_W^2)^2,\\
G^{Y(2)}_{WW} &= 2[2m_W^2 s^2 + s(2m_\chi^2 m_W^2 - 2m_\chi^4 - 11m_W^4) +\\
&\quad + 2m_W^2(m_\chi^4 + 4m_\chi^2 m_W^2 + m_W^4)],\\
G^{Y(3)}_{WW} &= m_\chi^2(m_W^2 - m_\chi^2)[s(m_\chi^2 + 3m_W^2) - 2m_W^2(2m_\chi^2 + m_W^2)],\\
G^{Y(4)}_{WW} &= (m_W^2 - m_\chi^2)[s(8m_W^4 - 4m_\chi^2 m_W^2) - m_\chi^2(8m_W^4 - m_\chi^4 - m_\chi^2 m_W^2)],
\end{aligned}$$

and

$$C^W_{\pm k} \equiv \left|C_V^{\chi_k^+\chi W^-}\right|^2 \pm \left|C_A^{\chi_k^+\chi W^-}\right|^2, \quad (4.83)$$

$$D^W_{\pm k} \equiv C_V^{\chi_k^+\chi W^-}\left(C_A^{\chi_k^+\chi W^-}\right)^* \pm C_A^{\chi_k^+\chi W^-}\left(C_V^{\chi_k^+\chi W^-}\right)^*; \quad (4.84)$$

Higgs (h, H)-chargino $(\chi_k^\pm)$ interference term:

$$\widetilde{w}_{WW}^{(h,H-\chi^\pm)} = \sum_{k=1}^{2} \mathrm{Re}\left[\left(\sum_{r=h,H}\frac{C^{WWr}C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r}\right)^*\right]\frac{1}{m_W^4}\Big[C^W_{+k}H_k^{(+)WW} + C^W_{-k}H_k^{(-)WW}\Big], \quad (4.85)$$

where

$$\begin{aligned}
H_k^{(+)WW} = {}& m_\chi\Big[s^2 + 2s(m^2_{\chi^\pm_k} - m_W^2 - m_\chi^2) + 8m_W^4 + 4m_W^2 m^2_{\chi^\pm_k} - 4m_W^2 m_\chi^2\Big]\\
&+2m_\chi\Big\{s^2 m^2_{\chi^\pm_k} + s\Big[2m_W^4 - 3m_W^2 m_\chi^2 - m^2_{\chi^\pm_k}m_W^2 + (m_\chi^2 - m^2_{\chi^\pm_k})^2\Big] -\\
&\quad - 4m_W^6 + 2m_W^4(m_\chi^2 + m^2_{\chi^\pm_k}) + 2m_W^2(m_\chi^2 - m^2_{\chi^\pm_k})^2\Big\} \times\\
&\times \mathcal{F}(s, m_\chi^2, m_W^2, m_W^2, m^2_{\chi^\pm_k}), \qquad (4.86)\\
H_k^{(-)WW} = {}& -m_{\chi^\pm_k}(s^2 - 2m_W^2 s) + m_{\chi^\pm_k}\Big[-(m_\chi^2 + m^2_{\chi^\pm_k})s^2 + 2s(2m_W^4 + m_W^2 m^2_{\chi^\pm_k} + 3m_W^2 m_\chi^2) -\\
&\quad - 24m_\chi^2 m_W^4\Big]\mathcal{F}(s, m_\chi^2, m_W^2, m_W^2, m^2_{\chi^\pm_k}); \qquad (4.87)
\end{aligned}$$

Z-chargino ($\chi_k^\pm$) interference term:

$$\begin{aligned}
\widetilde{w}_{WW}^{(Z-\chi^\pm)} &= \frac{1}{3}\sum_{k=1}^{2} \mathrm{Re}\left[\left(\frac{C^{WWZ}C_A^{\chi\chi Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right)^*\right] D_{+k}^W \times \\
&\times\Big\{s^3 + s^2(18m_W^2 - m_\chi^2 - 3m_{\chi_k^\pm}^2) + \\
&\quad + 2s\Big[-14m_W^4 - 18m_\chi^2 m_W^2 + 6m_{\chi_k^\pm}^2 m_W^2 - 3(m_\chi^2 - m_{\chi_k^\pm}^2)^2\Big] + \\
&\quad + 4m_W^2\Big[3(m_\chi^2 - m_{\chi_k^\pm}^2)^2 - m_W^2(11m_\chi^2 - 3m_{\chi_k^\pm}^2) - 6m_W^4\Big] + \\
&\quad + 6\Big[s^2 m_{\chi_k^\pm}^2(m_\chi^2 - m_{\chi_k^\pm}^2 + 4m_W^2) + s\Big[8m_W^6 - 5m_W^4(m_{\chi_k^\pm}^2 + 3m_\chi^2) + \\
&\qquad + 2m_W^2(3m_\chi^4 - 5m_\chi^2 m_{\chi_k^\pm}^2 + 2m_{\chi_k^\pm}^4) + (m_\chi^2 - m_{\chi_k^\pm}^2)^3\Big] + \\
&\qquad + m_W^2[4m_W^6 - 2m_W^4(3m_{\chi_k^\pm}^2 + 5m_\chi^2) + 8m_W^2 m_\chi^2(m_\chi^2 - m_{\chi_k^\pm}^2) + \\
&\qquad + 2(m_{\chi_k^\pm}^2 - m_\chi^2)^3]\Big]\mathcal{F}(s, m_\chi^2, m_W^2, m_W^2, m_{\chi_k^\pm}^2)\Big\}. \qquad (4.88)
\end{aligned}$$

4.9 $\chi\chi \to ZZ$

This process involves the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{w}_{ZZ} = \widetilde{w}_{ZZ}^{(h,H)} + \widetilde{w}_{ZZ}^{(\chi^0)} + \widetilde{w}_{ZZ}^{(h,H-\chi^0)} \ : \qquad (4.89)$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{w}_{ZZ}^{(h,H)} = \left|\sum_{r=h,H} \frac{C^{ZZr}C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r}\right|^2 (s - 4m_\chi^2)\frac{s^2 - 4m_Z^2 s + 12m_Z^4}{16m_Z^4}\,; \qquad (4.90)$$

Neutralino (χ_i^0) exchange:

$$\widetilde{w}_{ZZ}^{(\chi^0)} = \frac{1}{2m_Z^4}\sum_{i,j=1}^{4}\left[m_{\chi_i^0} m_{\chi_j^0} I_{ij}^{ZZ} + m_\chi m_{\chi_i^0} J_{ij}^{ZZ} + K_{ij}^{ZZ}\right], \qquad (4.91)$$

where

$$\begin{aligned}
I_{ij}^{ZZ} &= \Big\{(C_{-i}^{Z*}C_{-j}^Z + D_{-i}^{Z*}D_{-j}^Z) \times \\
&\times\Big[(s - 4m_Z^2)\mathcal{T}_2 - G_{ZZ}^{T(1)}\mathcal{T}_1 + G_{ZZ}^{T(2)}\mathcal{T}_0 + (s - 4m_Z^2)\mathcal{Y}_2 - G_{ZZ}^{Y(1)}\mathcal{Y}_0\Big] + \\
&+ (C_{-i}^{Z*}C_{-j}^Z - D_{-i}^{Z*}D_{-j}^Z)\Big[-18m_Z^4 m_\chi^2\mathcal{T}_0 - \\
&- m_\chi^2(-s^2 + 4m_Z^2 s + 6m_Z^4)\mathcal{Y}_0\Big]\Big\}(s, m_\chi^2, m_Z^2, m_Z^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \qquad (4.92) \\
J_{ij}^{ZZ} &= C_{-i}^{Z*}C_{+j}^Z\Big[12m_Z^2\mathcal{T}_2 + 12m_Z^2(m_Z^2 - 2m_\chi^2)\mathcal{T}_1 +
\end{aligned}$$

$$+ 12m_Z^2(m_\chi^4 + m_\chi^2 m_Z^2 - 2m_Z^4)\mathcal{T}_0 - 4(s-m_Z^2)\mathcal{Y}_2 + $$
$$+ 2m_Z^2(2s-3m_Z^2)\mathcal{Y}_1 + G_{ZZ}^{Y(2)}\mathcal{Y}_0\Big](s, m_\chi^2, m_Z^2, m_Z^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \quad (4.93)$$

$$\begin{aligned} K_{ij}^{ZZ} &= C_{+i}^{Z*}C_{+j}^{Z}\Big[-\mathcal{T}_4 - (s-2m_Z^2-4m_\chi^2)\mathcal{T}_3 - G_{ZZ}^{T(3)}\mathcal{T}_2 - G_{ZZ}^{T(4)}\mathcal{T}_1 - \\ &\quad - (m_Z^2-m_\chi^2)^2(2m_Z^2+m_\chi^2)\mathcal{T}_0 + \mathcal{Y}_4 + G_{ZZ}^{Y(3)}\mathcal{Y}_2 - \\ &\quad - G_{ZZ}^{Y(4)}\mathcal{Y}_0\Big](s, m_\chi^2, m_Z^2, m_Z^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)\,, \end{aligned} \tag{4.94}$$

and

$$\begin{aligned} G_{ZZ}^{T(1)} &= 2\left[s(m_\chi^2+2m_Z^2) - 4m_Z^2(m_\chi^2+m_Z^2)\right], \\ G_{ZZ}^{T(2)} &= s(m_\chi^2+2m_Z^2)^2 + m_Z^2(-4m_\chi^4 - 10m_\chi^2 m_Z^2 - 4m_Z^4)\,, \\ G_{ZZ}^{T(3)} &= -2s(m_\chi^2+2m_Z^2) + 6m_\chi^4 + 6m_\chi^2 m_Z^2 + 5m_Z^4\,, \\ G_{ZZ}^{T(4)} &= s(m_\chi^2+2m_Z^2)^2 - 8m_Z^6 + 18m_Z^4 m_\chi^2 - 6m_\chi^4 m_Z^2 - 4m_\chi^6\,, \\ G_{ZZ}^{Y(1)} &= -sm_\chi^2(m_\chi^2+2m_Z^2) + 6m_\chi^2 m_Z^4 + 4m_Z^2(m_\chi^2-m_Z^2)^2\,, \\ G_{ZZ}^{Y(2)} &= 2\left[2m_Z^2 s^2 + s(2m_\chi^2 m_Z^2 - 2m_\chi^4 - 11m_Z^4) + 2m_Z^2(m_\chi^4 + 4m_\chi^2 m_Z^2 + m_Z^4)\right], \\ G_{ZZ}^{Y(3)} &= s(m_\chi^2-4m_Z^2) + 2m_\chi^4 - 4m_\chi^2 m_Z^2 + m_Z^4\,, \\ G_{ZZ}^{Y(4)} &= (m_Z^2-m_\chi^2)\left[s(8m_Z^4 + m_\chi^4 - m_\chi^2 m_Z^2) + m_\chi^2(-10m_Z^4 + m_\chi^4 - 3m_\chi^2 m_Z^2)\right], \end{aligned}$$

and

$$C_{\pm i}^{Z} \equiv |C_V^{\chi_i^0\chi Z}|^2 \pm |C_A^{\chi_i^0\chi Z}|^2\,, \tag{4.95}$$
$$D_{\pm i}^{Z} \equiv C_V^{\chi_i^0\chi Z}\left(C_A^{\chi_i^0\chi Z}\right)^* \pm C_A^{\chi_i^0\chi Z}\left(C_V^{\chi_i^0\chi Z}\right)^*; \tag{4.96}$$

Higgs (h,H)-neutralino (χ_i^0) interference term:

$$\widetilde{w}_{ZZ}^{(h,H-\chi^0)} = \sum_{i=1}^{4}\mathrm{Re}\left[\left(\sum_{r=h,H}\frac{C^{ZZr}C_S^{\chi\chi r}}{s-m_r^2+i\Gamma_r m_r}\right)^*\right]\frac{1}{2m_Z^4}\Big[C_{+i}^Z H_i^{(+)ZZ} + C_{-i}^Z H_i^{(-)ZZ}\Big]\,, \tag{4.97}$$

where

$$\begin{aligned} H_i^{(+)ZZ} &= m_\chi\Big\{s^2 - 2s(m_\chi^2+m_Z^2-m_{\chi_i^0}^2) - 4m_Z^2(m_\chi^2-2m_Z^2-m_{\chi_i^0}^2) + \\ &\quad + 2\Big[s^2 m_{\chi_i^0}^2 + s\left\{(m_\chi^2-m_{\chi_i^0}^2)^2 - m_Z^2(3m_\chi^2-2m_Z^2+m_{\chi_i^0}^2)\right\} + \\ &\quad + 2m_Z^2\left\{(m_\chi^2-m_{\chi_i^0}^2)^2 + m_Z^2(m_\chi^2-2m_Z^2+m_{\chi_i^0}^2)\right\}\Big] \times \\ &\quad \times \mathcal{F}(s, m_\chi^2, m_Z^2, m_Z^2, m_{\chi_i^0}^2)\Big\}, \end{aligned} \tag{4.98}$$

$$\begin{aligned} H_i^{(-)ZZ} &= -m_{\chi_i^0}\Big\{s^2 - 2m_Z^2 s + \Big[s^2(m_\chi^2+m_{\chi_i^0}^2) - 2m_Z^2(2m_Z^2+3m_\chi^2+m_{\chi_i^0}^2)s + 24m_Z^4 m_\chi^2\Big] \times \\ &\quad \times \mathcal{F}(s, m_\chi^2, m_Z^2, m_Z^2, m_{\chi_i^0}^2)\Big\}. \end{aligned} \tag{4.99}$$

Note that $D_{+i}^Z = 0$, because $C_V^{\chi_i^0\chi_j^0 Z}$ is pure imaginary and $C_A^{\chi_i^0\chi_j^0 Z}$ is real.

4.10 $\chi\chi \to \bar{f}f$

This process involves the s-channel Higgs boson (h, H and A) and Z boson exchange and the t- and u-channel sfermion ($\widetilde{f}_a$) exchange

$$\widetilde{w}_{\bar{f}f} = \widetilde{w}_{\bar{f}f}^{(h,H)} + \widetilde{w}_{\bar{f}f}^{(A)} + \widetilde{w}_{\bar{f}f}^{(Z)} + \widetilde{w}_{\bar{f}f}^{(\widetilde{f})} + \widetilde{w}_{\bar{f}f}^{(h,H-\widetilde{f})} + \widetilde{w}_{\bar{f}f}^{(A-Z)} + \widetilde{w}_{\bar{f}f}^{(A-\widetilde{f})} + \widetilde{w}_{\bar{f}f}^{(Z-\widetilde{f})} \; : \tag{4.100}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{w}_{\bar{f}f}^{(h,H)} = \left| \sum_{r=h,H} \frac{C_S^{ffr} C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r} \right|^2 (s - 4m_\chi^2)(s - 4m_f^2)\,; \tag{4.101}$$

CP-odd Higgs-boson (A) exchange:

$$\widetilde{w}_{\bar{f}f}^{(A)} = \left| \frac{C_P^{ffA} C_P^{\chi\chi A}}{s - m_A^2 + i\Gamma_A m_A} \right|^2 s^2\,; \tag{4.102}$$

Z-boson exchange:

$$\begin{aligned}\widetilde{w}_{\bar{f}f}^{(Z)} = \frac{4}{3} \left| \frac{C_A^{\chi\chi Z}}{s - m_Z^2 + i\Gamma_Z m_Z} \right|^2 \Bigg\{ & 12|C_A^{ffZ}|^2 \frac{m_\chi^2 m_f^2}{m_Z^4}(s - m_Z^2)^2 + \\ & + \Big[|C_V^{ffZ}|^2(s + 2m_f^2) + |C_A^{ffZ}|^2(s - 4m_f^2)\Big](s - 4m_\chi^2)\Bigg\}\,;\end{aligned} \tag{4.103}$$

Sfermion $(\widetilde{f}_a)$ exchange:

$$\begin{aligned}\widetilde{w}_{\bar{f}f}^{(\widetilde{f})} = \frac{1}{4}\sum_{a,b} \Bigg[& (C_+^a C_+^b + D_+^a D_+^b)\Big[\mathcal{T}_2 - 2(m_\chi^2 + m_f^2)\mathcal{T}_1 + \Big\{(m_\chi^2 + m_f^2)^2 + 4m_\chi^2 m_f^2\Big\}\mathcal{T}_0\Big] + \\ & + (C_-^a C_-^b - D_-^a D_-^b)\Big[\mathcal{T}_2 - 2(m_\chi^2 + m_f^2)\mathcal{T}_1 + (m_\chi^2 - m_f^2)^2\mathcal{T}_0\Big] - \\ & - (C_+^a C_+^b - D_-^a D_-^b) 4m_\chi^2 m_f^2 \mathcal{Y}_0 + \\ & + (C_+^a C_+^b + C_-^a C_-^b) s m_\chi^2 \mathcal{Y}_0 + (C_+^a C_+^b - C_-^a C_-^b) s m_f^2 \mathcal{Y}_0 + \\ & + (D_+^a D_+^b - D_-^a D_-^b)\Big[\mathcal{Y}_2 + (m_\chi^2 + m_f^2)^2\mathcal{Y}_0\Big] + \\ & + (C_+^a D_+^b + D_+^a C_+^b) m_\chi m_f \Big[4(m_\chi^2 + m_f^2)\mathcal{T}_0 - 4\mathcal{T}_1 + s\mathcal{Y}_0\Big] - \\ & - (C_+^a D_+^b - D_+^a C_+^b) m_\chi m_f \mathcal{Y}_1\Bigg](s, m_\chi^2, m_f^2, m_f^2, m_{\widetilde{f}_a}^2, m_{\widetilde{f}_b}^2)\,,\end{aligned} \tag{4.104}$$

where

$$C_\pm^a = |\Lambda_{fL}^a|^2 \pm |\Lambda_{fR}^a|^2\,, \tag{4.105}$$

$$D_\pm^a = \Lambda_{fL}^a (\Lambda_{fR}^a)^* \pm (\Lambda_{fL}^a)^* \Lambda_{fR}^a\,. \tag{4.106}$$

In the above, a is the index for sfermion mass eigenstates so that $a = 1, \ldots, 6$ for squarks and charged sleptons, and $a = 1, 2, 3$ for sneutrinos.

The symbol f represents each fermion: $f = u, c, t, \ldots$ The symbol $\widetilde{f}_a$ should be understood as follows. For up-type quarks, down-type quarks and charged leptons, the corresponding symbol $\widetilde{f}_a$ represents $\widetilde{u}_a$, $\widetilde{d}_a$ and $\widetilde{e}_a$ $(a = 1, \ldots, 6)$, respectively. For neutrinos, $\widetilde{f}_a$ represents $\widetilde{\nu}_a$ $(a = 1, 2, 3)$. For e.g. for $\widetilde{w}_{\bar{c}c}^{(\widetilde{u})}$, the last argument at the end of eq. (4.104) should read $(s, m_\chi^2, m_c^2, m_c^2, m_{\widetilde{u}_a}^2, m_{\widetilde{u}_b}^2)$ when $a = 1, \ldots, 6$.

The coupling Λ_{fL}^a for each fermion is defined by $\Lambda_{nai}^{(f)L}$ in appendix A as $\Lambda_{uL}^a = \Lambda_{1a1}^{(u)L}$, $\Lambda_{cL}^a = \Lambda_{2a1}^{(u)L}$, $\Lambda_{tL}^a = \Lambda_{3a1}^{(u)L}$, $\Lambda_{dL}^a = \Lambda_{1a1}^{(d)L}$, $\Lambda_{sL}^a = \Lambda_{2a1}^{(d)L}$, $\Lambda_{bL}^a = \Lambda_{3a1}^{(d)L}$, etc. Similarly, the coupling Λ_{fR}^a is defined by $\Lambda_{nai}^{(f)R}$.

Higgs (h, H)-sfermion $(\widetilde{f}_a)$ interference term:

$$\begin{aligned}\widetilde{w}_{\bar{f}f}^{(h,H-\widetilde{f})} &= \sum_a \mathrm{Re}\left[\sum_{r=h,H} \frac{C_S^{ffr} C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r}\right] \times \\ &\quad \times \Big[C_+^a 2 m_\chi m_f \Big\{2 + \Big[s - 2(m_\chi^2 + m_f^2 - m_{\widetilde{f}_a}^2)\Big] \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\widetilde{f}_a}^2)\Big\} + \\ &\quad + D_+^a \Big\{s + \Big[s(m_\chi^2 + m_f^2 + m_{\widetilde{f}_a}^2) - 8 m_\chi^2 m_f^2\Big] \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\widetilde{f}_a}^2)\Big\}\Big];\end{aligned} \tag{4.107}$$

Higgs (A)-Z interference term:

$$\widetilde{w}_{\bar{f}f}^{(A-Z)} = \mathrm{Re}\left[\left(\frac{C_P^{ffA} C_P^{\chi\chi A}}{s - m_A^2 + i\Gamma_A m_A}\right)^* \left(\frac{C_A^{ffZ} C_A^{\chi\chi Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right)\right] \frac{8 m_\chi m_f}{m_Z^2} s(m_Z^2 - s); \tag{4.108}$$

Higgs (A)-sfermion $(\widetilde{f}_a)$ interference term:

$$\begin{aligned}\widetilde{w}_{\bar{f}f}^{(A-\widetilde{f})} &= -\sum_a \mathrm{Re}\left[\frac{C_P^{ffA} C_P^{\chi\chi A}}{s - m_A^2 + i\Gamma_A m_A}\right] \times \\ &\quad \times \Big[C_+^a 2 m_\chi m_f s \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\widetilde{f}_a}^2) + \\ &\quad + D_+^a \Big\{-s + s(m_\chi^2 + m_f^2 - m_{\widetilde{f}_a}^2) \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\widetilde{f}_a}^2)\Big\}\Big];\end{aligned} \tag{4.109}$$

Z-sfermion $(\widetilde{f}_a)$ interference term:

$$\widetilde{w}_{\bar{f}f}^{(Z-\widetilde{f})} = \sum_a \left\{\mathrm{Re}\left[\frac{C_A^{ffZ} C_A^{\chi\chi Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right] Z_a^{(+)\bar{f}f} + \mathrm{Re}\left[\frac{C_V^{ffZ} C_A^{\chi\chi Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right] Z_a^{(-)\bar{f}f}\right\}, \tag{4.110}$$

where

$$\begin{aligned}Z_a^{(+)\bar{f}f} &= C_+^a \Big\{s + 2(m_\chi^2 + m_f^2 - m_{\widetilde{f}_a}^2) - \\ &\quad - \Big[-2s(m_\chi^2 + m_f^2) + 2(m_\chi^2 + m_f^2 - m_{\widetilde{f}_a}^2)^2 + 8 m_\chi^2 m_f^2 \Big(2 - \frac{s}{m_Z^2}\Big)\Big] \times \\ &\quad \times \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\widetilde{f}_a}^2)\Big\}\end{aligned}$$

$$
- D_+^a 4m_\chi m_f \left\{ \frac{s}{m_Z^2} - 3 - \left[s + \left(\frac{s}{m_Z^2} - 3 \right)(m_\chi^2 + m_f^2 - m_{\tilde{f}_a}^2) \right] \times \right.
$$

$$
\left. \times \, \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\tilde{f}_a}^2) \right\} , \tag{4.111}
$$

$$
Z_a^{(-)\bar{f}f} = C_-^a \left\{ s + 2(m_\chi^2 + m_f^2 - m_{\tilde{f}_a}^2) + \left[2s(m_\chi^2 - m_f^2) - 2(m_\chi^2 + m_f^2 - m_{\tilde{f}_a}^2)^2 + 8m_\chi^2 m_f^2 \right] \times \right.
$$

$$
\left. \times \, \mathcal{F}(s, m_\chi^2, m_f^2, m_f^2, m_{\tilde{f}_a}^2) \right\} . \tag{4.112}
$$

This completes the list of all the tree-level two-body neutralino pair-annihilation channels in the MSSM.

5. Partial wave expansion

In the literature one still often uses the usual approximation in terms of the expansion in powers of x (or, equivalently, WIMP velocity-square), $\langle \sigma v_{\text{Møl}} \rangle \simeq a + bx$. This is because in the early days it was often much easier to compute the coefficients a and b, rather than the cross section itself [34]. Furthermore, the partial wave expansion gives a rather good approximation to the exact result but only far enough from s-channel resonances and thresholds for new final states. (For a recent detailed study, see ref. [19].)

The expansion of $\langle \sigma v_{\text{Møl}} \rangle$, as defined in eq. (2.2), is given by [21, 16]

$$
\langle \sigma v_{\text{Møl}} \rangle = \frac{1}{m_\chi^2} \left[w - \frac{3}{2} \left(2w - w' \right) x + \mathcal{O}(x^2) \right]_{s=4m_\chi^2} \equiv a + bx + \mathcal{O}(x^2) \, , \tag{5.1}
$$

where $w'(s)$ denotes $dw(s)/d(s/4m_\chi^2)$.

Analogously to the function $w(s)$ (eq. (4.3)), the coefficients a and b need to be summed over all possible final states $f_1 f_2$. One can write them as [21]

$$
a = \sum_{f_1 f_2} c\theta \left(4m_\chi^2 - (m_{f_1} + m_{f_2})^2 \right) v_{f_1 f_2} \widetilde{a}_{f_1 f_2} \, , \tag{5.2}
$$

$$
b = \sum_{f_1 f_2} c\theta \left(4m_\chi^2 - (m_{f_1} + m_{f_2})^2 \right) v_{f_1 f_2} \times
$$

$$
\times \left\{ \widetilde{b}_{f_1 f_2} + \widetilde{a}_{f_1 f_2} \left[-3 + \frac{3}{4} v_{f_1 f_2}^{-2} \left(\frac{m_{f_1}^2 + m_{f_2}^2}{2m_\chi^2} + \frac{(m_{f_1}^2 - m_{f_2}^2)^2}{8m_\chi^4} \right) \right] \right\} , \tag{5.3}
$$

where the summation extends over all possible two-body final states $f_1 f_2$, the coefficient c is defined in eq. (4.4), and

$$
v_{f_1 f_2} = \beta_f(4m_\chi^2, m_{f_1}, m_{f_2}) \, , \tag{5.4}
$$

and the velocity $\beta_f(s, m_{f_1}, m_{f_2})$ was defined in eq. (4.5). The "reduced" coefficients $\widetilde{a}_{f_1 f_2}$ and $\widetilde{b}_{f_1 f_2}$ are given by

$$
\widetilde{a}_{f_1 f_2} = \frac{1}{32\pi m_\chi^2} \widetilde{w}_{f_1 f_2}(4m_\chi^2) \, , \tag{5.5}
$$

$$
\widetilde{b}_{f_1 f_2} = \frac{3}{64\pi m_\chi^2} \widetilde{w}'_{f_1 f_2}(4m_\chi^2) \, , \tag{5.6}
$$

where $\widetilde{w}(s)$ was defined in eq. (4.6) and $\widetilde{w}'(s) \equiv d\widetilde{w}(s)/d(s/4m_\chi^2)$.

In this section, we provide a set of expressions for the coefficients a and b in the case of equal-mass final states. Using eq. (5.1), we have derived the coefficients for all the final states by using the analytic expressions for $w(s)$ presented in the previous section. In the case of unequal masses of the final state particles the resulting formulae are exceedingly lengthly and we will not include them here.

In the literature one can find several analytic formulae for the expansion coefficients, including [12, 13, 14, 2], but, due to different conventions and complexity of expressions, comparison is not always doable. We have checked our results for the a-coefficients in appropriate limits against published results and agreed in all cases.

All the couplings and auxiliary functions appearing below are listed in the appendices.

5.1 $\chi\chi \to hh$

This process involves the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{a}_{hh} = \widetilde{a}_{hh}^{(h,H)} + \widetilde{a}_{hh}^{(\chi^0)} + \widetilde{a}_{hh}^{(h,H-\chi^0)}\,, \tag{5.7}$$

$$\widetilde{b}_{hh} = \widetilde{b}_{hh}^{(h,H)} + \widetilde{b}_{hh}^{(\chi^0)} + \widetilde{b}_{hh}^{(h,H-\chi^0)}\,: \tag{5.8}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{a}_{hh}^{(h,H)} = 0\,, \tag{5.9}$$

$$\widetilde{b}_{hh}^{(h,H)} = \frac{3}{64\pi}\left|\sum_{r=h,H} \frac{C^{hhr} C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r}\right|^2; \tag{5.10}$$

Neutralino (χ_i^0) exchange:

$$\widetilde{a}_{hh}^{(\chi^0)} = 0\,, \tag{5.11}$$

$$\begin{aligned}\widetilde{b}_{hh}^{(\chi^0)} &= \frac{1}{16\pi}\sum_{i,j=1}^{4} (C_S^{\chi_i^0\chi h})^2 (C_S^{\chi_j^0\chi h *})^2 \frac{1}{\Delta_{hi}^2\Delta_{hj}^2} \times \\ &\quad \times\Big[4m_\chi^2(m_\chi^2 - m_h^2)^2 + 4m_\chi(m_\chi^2 - m_h^2)(m_\chi + m_{\chi_i^0})\Delta_{hi} + \\ &\qquad + 3(m_\chi + m_{\chi_i^0})(m_\chi + m_{\chi_j^0})\Delta_{hi}\Delta_{hj}\Big]\,,\end{aligned} \tag{5.12}$$

where $\Delta_{hi} \equiv m_h^2 - m_\chi^2 - m_{\chi_i^0}^2$.

Higgs (h, H)-neutralino (χ_i^0) interference term:

$$\widetilde{a}_{hh}^{(h,H-\chi^0)} = 0\,, \tag{5.13}$$

$$\begin{aligned}\widetilde{b}_{hh}^{(h,H-\chi^0)} &= \frac{1}{16\pi}\sum_{i=1}^{4} \mathrm{Re}\left[\sum_{r=h,H}\left(\frac{C^{hhr} C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r}\right)^* C_S^{\chi_i^0\chi h} C_S^{\chi_i^0\chi h}\right] \times \\ &\quad \times \frac{[2m_\chi(m_\chi^2 - m_h^2) + 3(m_\chi + m_{\chi_i^0})\Delta_{hi}]}{\Delta_{hi}^2}\,.\end{aligned} \tag{5.14}$$

The expressions for HH final state are obtained from the above by replacing m_h, C^{hhr}, $C_S^{\chi_i^0\chi h}$, $C_S^{\chi_j^0\chi h}$ with m_H, C^{HHr}, $C_S^{\chi_i^0\chi H}$, $C_S^{\chi_j^0\chi H}$, respectively.

5.2 $\chi\chi \to AA$

Similarly to the final state hh, this process proceeds via the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$\widetilde{a}_{AA} = \widetilde{a}_{AA}^{(h,H)} + \widetilde{a}_{AA}^{(\chi^0)} + \widetilde{a}_{AA}^{(h,H-\chi^0)}\,, \tag{5.15}$$

$$\widetilde{b}_{AA} = \widetilde{b}_{AA}^{(h,H)} + \widetilde{b}_{AA}^{(\chi^0)} + \widetilde{b}_{AA}^{(h,H-\chi^0)}\; : \tag{5.16}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{a}_{AA}^{(h,H)} = 0\,, \tag{5.17}$$

$$\widetilde{b}_{AA}^{(h,H)} = \frac{3}{64\pi} \left| \sum_{r=h,H} \frac{C^{AAr}\, C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r} \right|^2 ; \tag{5.18}$$

Neutralino (χ_i^0) exchange:

$$\widetilde{a}_{AA}^{(\chi^0)} = 0\,, \tag{5.19}$$

$$\begin{aligned}
\widetilde{b}_{AA}^{(\chi^0)} = \frac{1}{16\pi} \sum_{i,j=1}^{4} & (C_P^{\chi_i^0 \chi A})^2 (C_P^{\chi_j^0 \chi A *})^2 \frac{1}{\Delta_{Ai}^2 \Delta_{Aj}^2} \times \\
& \times \Big[4m_\chi^2 (m_\chi^2 - m_A^2)^2 + 4m_\chi (m_\chi^2 - m_A^2)(m_\chi - m_{\chi_i^0})\Delta_{Ai} + \\
& + 3(m_\chi - m_{\chi_i^0})(m_\chi - m_{\chi_j^0})\Delta_{Ai}\Delta_{Aj} \Big]\,,
\end{aligned} \tag{5.20}$$

where $\Delta_{Ai} \equiv m_A^2 - m_\chi^2 - m_{\chi_i^0}^2$.

Higgs (h, H)-neutralino (χ_i^0) interference term:

$$\widetilde{a}_{AA}^{(h,H-\chi^0)} = 0\,, \tag{5.21}$$

$$\begin{aligned}
\widetilde{b}_{AA}^{(h,H-\chi^0)} = & \frac{1}{16\pi} \sum_{i=1}^{4} \mathrm{Re} \left[\sum_{r=h,H} \left(\frac{C^{AAr} C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r} \right)^* (C_P^{\chi_i^0 \chi A})^2 \right] \times \\
& \times \frac{[-2m_\chi (m_\chi^2 - m_A^2) - 3(m_\chi - m_{\chi_i^0})\Delta_{Ai}]}{\Delta_{Ai}^2}\,.
\end{aligned} \tag{5.22}$$

5.3 $\chi\chi \to H^+H^-$

This process proceeds via the s-channel Z and CP-even Higgs boson (h, H) exchange as well as the t- and u-channel chargino ($\chi_k^\pm$, $k = 1, 2$) exchange

$$\widetilde{a}_{H^+H^-} = \widetilde{a}_{H^+H^-}^{(h,H)} + \widetilde{a}_{H^+H^-}^{(Z)} + \widetilde{a}_{H^+H^-}^{(\chi^\pm)} + \widetilde{a}_{H^+H^-}^{(h,H-\chi^\pm)} + \widetilde{a}_{H^+H^-}^{(Z-\chi^\pm)}\,, \tag{5.23}$$

$$\widetilde{b}_{H^+H^-} = \widetilde{b}_{H^+H^-}^{(h,H)} + \widetilde{b}_{H^+H^-}^{(Z)} + \widetilde{b}_{H^+H^-}^{(\chi^\pm)} + \widetilde{b}_{H^+H^-}^{(h,H-\chi^\pm)} + \widetilde{b}_{H^+H^-}^{(Z-\chi^\pm)}\; : \tag{5.24}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{a}_{H^+H^-}^{(h,H)} = 0\,, \tag{5.25}$$

$$\widetilde{b}_{H^+H^-}^{(h,H)} = \frac{3}{32\pi} \left| \sum_{r=h,H} \frac{C^{H^+H^-r} C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r} \right|^2 ; \tag{5.26}$$

Z-boson exchange:

$$\widetilde{a}^{(Z)}_{H^+H^-} = 0\,, \tag{5.27}$$
$$\widetilde{b}^{(Z)}_{H^+H^-} = \frac{1}{4\pi}\left|\frac{C^{H^+H^-Z}C^{\chi\chi Z}_A}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z}\right|^2 (m_\chi^2 - m_{H^\pm}^2)\,; \tag{5.28}$$

Chargino ($\chi_k^\pm$) exchange:

$$\widetilde{a}^{(\chi^\pm)}_{H^+H^-} = 0\,, \tag{5.29}$$
$$\begin{aligned}\widetilde{b}^{(\chi^\pm)}_{H^+H^-} = \frac{1}{8\pi}\sum_{k,l=1}^{2}\frac{1}{\Delta^2_{H^\pm k}\Delta^2_{H^\pm l}}\Big\{ & C^H_{+k}C^H_{+l}m_\chi^2\Big[4(m_\chi^2 - m_{H^\pm}^2)^2 + 4(m_\chi^2 - m_{H^\pm}^2)\Delta_{H^\pm k} \\ & \quad + 3\Delta_{H^\pm k}\Delta_{H^\pm l}\Big] + \\ & + D^H_{-k}D^H_{-l}\Big[4m_\chi^2 m_{\chi_k^\pm}m_{\chi_l^\pm}(m_\chi^2 - m_{H^\pm}^2)\Big] + \\ & + C^H_{+k}C^H_{-l}2m_\chi m_{\chi_k^\pm}\Big[2(m_\chi^2 - m_{H^\pm}^2)\Delta_{H^\pm k} + 3\Delta^2_{H^\pm k}\Big] + \\ & + C^H_{-k}C^H_{-l}\Big[3m_{\chi_k^\pm}m_{\chi_l^\pm}\Delta_{H^\pm k}\Delta_{H^\pm l}\Big] + \\ & + D^H_{+k}D^H_{+l}\Big[2(m_\chi^2 - m_{H^\pm}^2)\Delta_{H^\pm k}\Delta_{H^\pm l}\Big]\Big\}\,, \end{aligned} \tag{5.30}$$

where $\Delta_{H^\pm k} \equiv m_{H^\pm}^2 - m_\chi^2 - m_{\chi_k^\pm}^2$. The couplings $C^H_{\pm k}$ and $D^H_{\pm k}$ are given in eqs. (4.38) and (4.39).

Higgs (h, H)-chargino ($\chi_k^\pm$) interference term:

$$\widetilde{a}^{(h,H-\chi^\pm)}_{H^+H^-} = 0\,, \tag{5.31}$$
$$\begin{aligned}\widetilde{b}^{(h,H-\chi^\pm)}_{H^+H^-} = & \frac{1}{8\pi}\sum_{k=1}^{2}\mathrm{Re}\left[\sum_{r=h,H}\left(\frac{C^{H^+H^-r}C^{\chi\chi r}_S}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r}\right)^*\right]\frac{1}{\Delta^2_{H^\pm}}\times \\ & \times\Big\{C^H_{+k}m_\chi\Big[2(m_\chi^2 - m_{H^\pm}^2) + 3\Delta_{H_k^\pm}\Big] + C^H_{-k}3m_{\chi_k^\pm}\Delta_{H_k^\pm}\Big\}\,; \end{aligned} \tag{5.32}$$

Z-chargino ($\chi_k^\pm$) interference term:

$$\widetilde{a}^{(Z-\chi^\pm)}_{H^+H^-} = 0\,, \tag{5.33}$$
$$\widetilde{b}^{(Z-\chi^\pm)}_{H^+H^-} = -\frac{1}{2\pi}\sum_{k=1}^{2}\mathrm{Re}\left[\left(\frac{C^{H^+H^-Z}C^{\chi\chi Z}_A}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z}\right)^* D^H_{+k}\right]\frac{(m_\chi^2 - m_{H^\pm}^2)}{\Delta_{H^\pm k}}\,. \tag{5.34}$$

5.4 $\chi\chi \to WW$

This process involves the s-channel CP-even Higgs boson (h and H) and Z exchange, and the t- and u-channel chargino ($\chi_k^\pm$, $k = 1, 2$) exchange

$$\widetilde{a}_{WW} = \widetilde{a}^{(h,H)}_{WW} + \widetilde{a}^{(Z)}_{WW} + \widetilde{a}^{(\chi^\pm)}_{WW} + \widetilde{a}^{(h,H-\chi^\pm)}_{WW} + \widetilde{a}^{(Z-\chi^\pm)}_{WW}\,, \tag{5.35}$$
$$\widetilde{b}_{WW} = \widetilde{b}^{(h,H)}_{WW} + \widetilde{b}^{(Z)}_{WW} + \widetilde{b}^{(\chi^\pm)}_{WW} + \widetilde{b}^{(h,H-\chi^\pm)}_{WW} + \widetilde{b}^{(Z-\chi^\pm)}_{WW}\,; \tag{5.36}$$

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CP-even Higgs-boson (h, H) exchange:

$$\widetilde{a}_{WW}^{(h,H)} = 0\,, \tag{5.37}$$

$$\widetilde{b}_{WW}^{(h,H)} = \frac{3}{32\pi}\left|\sum_{r=h,H}\frac{C^{WWr}C_S^{\chi\chi r}}{s-m_r^2+i\Gamma_r m_r}\right|^2 \frac{3m_W^4-4m_W^2m_\chi^2+4m_\chi^4}{m_W^4}\,; \tag{5.38}$$

Z-boson exchange:

$$\widetilde{a}_{WW}^{(Z)} = 0\,, \tag{5.39}$$

$$\widetilde{b}_{WW}^{(Z)} = \frac{1}{4\pi}\left|\frac{C^{WWZ}C_A^{\chi\chi Z}}{s-m_Z^2+i\Gamma_Z m_Z}\right|^2 \frac{(3m_W^4+20m_W^2m_\chi^2+4m_\chi^4)(m_\chi^2-m_W^2)}{m_W^4}\,; \tag{5.40}$$

Chargino $(\chi_k^\pm)$ exchange:

$$\begin{aligned}
\widetilde{a}_{WW}^{\chi^\pm} = \frac{1}{4\pi}\sum_{k,l=1}^{2}\frac{1}{m_W^4\Delta_{Wk}\Delta_{Wl}}\Big\{ & C_{+k}^{W*}C_{+l}^{W}\Big[2m_W^4(m_\chi^2-m_W^2)\Big] + \\
& + D_{+k}^{W*}D_{+l}^{W}\Big[m_{\chi_k^\pm}m_{\chi_l^\pm}(4m_\chi^4+3m_W^4-4m_W^2m_\chi^2)\Big]\Big\}\,,
\end{aligned} \tag{5.41}$$

$$\begin{aligned}
\widetilde{b}_{WW}^{\chi^\pm} = \frac{1}{8\pi}\sum_{k,l=1}^{2}\frac{1}{m_W^4\Delta_{Wk}^3\Delta_{Wl}^3}\times & \\
\times\Big\{ & C_{+k}^{W*}C_{+l}^{W}\Big[16m_\chi^2m_W^4(m_\chi^2-m_W^2)^2\Delta_{Wk}^2 + \\
& \quad + 4m_\chi^2m_W^2(3m_W^2+4m_\chi^2)(m_\chi^2-m_W^2)^2\Delta_{Wk}\Delta_{Wl} - \\
& \quad - 4m_\chi^2m_W^2(7m_W^4+m_\chi^2m_W^2-8m_\chi^4)\Delta_{Wk}^2\Delta_{Wl} + \\
& \quad + (8m_W^6+3m_W^4m_\chi^2+8m_W^2m_\chi^4+8m_\chi^6)\Delta_{Wk}^2\Delta_{Wl}^2\Big] + \\
& + C_{+k}^{W*}C_{-l}^{W}\Big[48m_W^2m_\chi^3m_{\chi_k^\pm}(m_\chi^2-m_W^2)^2\Delta_{Wk}\Delta_{Wl} + \\
& \quad + 4m_W^2m_\chi m_{\chi_k^\pm}(3m_W^4-5m_W^2m_\chi^2+2m_\chi^4)\Delta_{Wk}^2\Delta_{Wl} + \\
& \quad + 8m_W^2m_\chi m_{\chi_k^\pm}(3m_W^4-8m_W^2m_\chi^2+5m_\chi^4)\Delta_{Wk}\Delta_{Wl}^2 + \\
& \quad + 2m_\chi m_{\chi_k^\pm}(-5m_W^4+10m_W^2m_\chi^2-8m_\chi^4)\Delta_{Wk}^2\Delta_{Wl}^2\Big] + \\
& + C_{-k}^{W*}C_{-l}^{W}\Big[16m_\chi^2m_{\chi_k^\pm}m_{\chi_l^\pm}(m_\chi^2-m_W^2)^2(m_W^2+m_\chi^2)\Delta_{Wk}\Delta_{Wl} + \\
& \quad + 8m_\chi^2m_{\chi_k^\pm}m_{\chi_l^\pm}(m_W^4-3m_W^2m_\chi^2+2m_\chi^4)\Delta_{Wk}^2\Delta_{Wl} + \\
& \quad + 3m_{\chi_k^\pm}m_{\chi_l^\pm}(3m_W^4-4m_W^2m_\chi^2+4m_\chi^4)\Delta_{Wk}^2\Delta_{Wl}^2\Big] + \\
& + D_{+k}^{W*}D_{+l}^{W}\Big[16m_\chi^2m_W^2(m_\chi^2-m_W^2)^3\Delta_{Wk}\Delta_{Wl} + \\
& \quad + 32m_\chi^2m_W^2(m_\chi^2-m_W^2)^2\Delta_{Wk}^2\Delta_{Wl} + \\
& \quad + 2(-3m_W^6-17m_W^4m_\chi^2+16m_W^2m_\chi^4+4m_\chi^6)\Delta_{Wk}^2\Delta_{Wl}^2\Big] + \\
& + D_{-k}^{W*}D_{-l}^{W}\Big[8m_\chi^2m_{\chi_k^\pm}m_{\chi_l^\pm}(-3m_W^6+7m_W^4m_\chi^2-8m_W^2m_\chi^4+4m_\chi^6)\Delta_{Wk}^2 -
\end{aligned}$$

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$$
\begin{aligned}
&- 8m_W^2 m_\chi^2 m_{\chi_k^\pm} m_{\chi_l^\pm}(m_W^4 + m_W^2 m_\chi^2 - 2m_\chi^4)\Delta_{Wk}\Delta_{Wl} + \\
&+ 4m_\chi^2 m_{\chi_k^\pm} m_{\chi_l^\pm}(11m_W^4 - 18m_W^2 m_\chi^2 + 16m_\chi^4)\Delta_{Wk}^2\Delta_{Wl} + \\
&+ 3m_{\chi_k^\pm} m_{\chi_l^\pm}(3m_W^4 - 8m_W^2 m_\chi^2 + 12m_\chi^4)\Delta_{Wk}^2\Delta_{Wl}^2\Big]\Big\}\,,
\end{aligned} \tag{5.42}
$$

where $\Delta_{Wk} \equiv m_W^2 - m_\chi^2 - m_{\chi_k^\pm}^2$. The couplings $C_{\pm k}^W$ and $D_{\pm k}^W$ are given in eqs. (4.83) and (4.84).

Higgs (h, H)-chargino $(\chi_k^\pm)$ interference term:

$$
\widetilde{a}_{WW}^{(h,H-\chi^\pm)} = 0\,, \tag{5.43}
$$

$$
\begin{aligned}
\widetilde{b}_{WW}^{(h,H-\chi^\pm)} &= \frac{1}{8\pi}\sum_{k=1}^{2}\mathrm{Re}\left[\left(\sum_{r=h,H}\frac{C^{WWr}C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r}\right)^*\right]\frac{1}{m_W^4\Delta_{Wk}^2}\times \\
&\times\Big\{C_{+k}^W\Big[2m_\chi m_W^2(3m_W^4 - 5m_W^2 m_\chi^2 + 2m_\chi^4) + \\
&\qquad + m_\chi(-5m_W^4 + 10m_W^2 m_\chi^2 - 8m_\chi^4)\Delta_{Wk}\Big] + \\
&\quad + C_{-k}^W\Big[4m_\chi^2 m_{\chi_k^\pm}(m_W^4 - 3m_W^2 m_\chi^2 + 2m_\chi^4) + \\
&\qquad + 3m_{\chi_k^\pm}(3m_W^4 - 4m_W^2 m_\chi^2 + 4m_\chi^4)\Delta_{Wk}\Big]\Big\}\,;
\end{aligned} \tag{5.44}
$$

Z-chargino $(\chi_k^\pm)$ interference term:

$$
\widetilde{a}_{WW}^{(Z-\chi^\pm)} = 0\,, \tag{5.45}
$$

$$
\begin{aligned}
\widetilde{b}_{WW}^{(Z-\chi^\pm)} &= \frac{1}{2\pi}\sum_{k=1}^{2}\mathrm{Re}\left[\left(\frac{C^{WWZ}C_A^{\chi\chi Z}}{s - m_Z^2 + i\Gamma_Z m_Z}\right)^*\right]\frac{1}{m_W^4\Delta_{Wk}^2}D_{+k}^W\times \\
&\times\Big[-8m_W^2 m_\chi^2(m_W^2 - m_\chi^2)^2 + (m_W^2 - m_\chi^2)(3m_W^4 + 20m_W^2 m_\chi^2 + 4m_\chi^4)\Delta_{Wk}\Big]\,.
\end{aligned} \tag{5.46}
$$

5.5 $\chi\chi \to ZZ$

This process involves the s-channel CP-even Higgs boson (h and H) exchange and the t- and u-channel neutralino (χ_i^0, $i = 1, \ldots, 4$) exchange

$$
\widetilde{a}_{ZZ} = \widetilde{a}_{ZZ}^{(h,H)} + \widetilde{a}_{ZZ}^{(\chi^0)} + \widetilde{a}_{ZZ}^{(h,H-\chi^0)}\,, \tag{5.47}
$$

$$
\widetilde{b}_{ZZ} = \widetilde{b}_{ZZ}^{(h,H)} + \widetilde{b}_{ZZ}^{(\chi^0)} + \widetilde{b}_{ZZ}^{(h,H-\chi^0)}\,; \tag{5.48}
$$

CP-even Higgs-boson (h, H) exchange:

$$
\widetilde{a}_{ZZ}^{(h,H)} = 0\,, \tag{5.49}
$$

$$
\widetilde{b}_{ZZ}^{(h,H)} = \frac{3}{64\pi}\left|\sum_{r=h,H}\frac{C^{ZZr}C_S^{\chi\chi r}}{s - m_r^2 + i\Gamma_r m_r}\right|^2\frac{3m_Z^4 - 4m_Z^2 m_\chi^2 + 4m_\chi^4}{m_Z^4}\,; \tag{5.50}
$$

Neutralino (χ_i^0) exchange:

$$\widetilde{a}_{ZZ}^{(\chi^0)} = \frac{1}{4\pi}\sum_{i,j=1}^{4} C_{+i}^{Z*}C_{+j}^{Z}\frac{(m_\chi^2-m_Z^2)}{\Delta_{Zi}\Delta_{Zj}}\,, \tag{5.51}$$

$$\widetilde{b}_{ZZ}^{(\chi^0)} = \frac{1}{16\pi}\sum_{i,j=1}^{4}\frac{1}{m_Z^4\Delta_{Zi}^3\Delta_{Zj}^3}\times$$
$$\times C_{+i}^{Z*}C_{+j}^{Z}\Big[D_{ij}^{(1)}\Delta_{Zi}^2 + D_{ij}^{(2)}\Delta_{Zi}\Delta_{Zj} + D_{ij}^{(3)}\Delta_{Zi}^2\Delta_{Zj} + D_{ij}^{(4)}\Delta_{Zi}^2\Delta_{Zj}^2\Big]\,, \tag{5.52}$$

where

$$D_{ij}^{(1)} = 16m_\chi^2 m_Z^4(m_Z^2-m_\chi^2)^2\,, \tag{5.53}$$

$$D_{ij}^{(2)} = 4m_\chi^2(m_Z^2-m_\chi^2)^2\Big\{3m_Z^4 + 4m_\chi^2 m_{\chi_i^0}m_{\chi_j^0} + $$
$$+ m_Z^2[4m_\chi^2 + 4m_{\chi_i^0}m_{\chi_j^0} - 6m_\chi(m_{\chi_i^0}+m_{\chi_j^0})]\Big\}\,, \tag{5.54}$$

$$D_{ij}^{(3)} = -4m_\chi(m_Z^2-m_\chi^2)\Big\{4m_\chi^3 m_{\chi_i^0}m_{\chi_j^0} + m_Z^4(7m_\chi + 3m_{\chi_i^0} + 6m_{\chi_j^0}) + $$
$$+ 2m_Z^2 m_\chi[4m_\chi^2 - m_{\chi_i^0}m_{\chi_j^0} - m_\chi(m_{\chi_i^0}+5m_{\chi_j^0})]\Big\}\,, \tag{5.55}$$

$$D_{ij}^{(4)} = 8m_Z^6 + 2m_Z^2 m_\chi^2[4m_\chi^2 - 6m_{\chi_i^0}m_{\chi_j^0} - 5m_\chi(m_{\chi_i^0}+m_{\chi_j^0})] + $$
$$+ 4m_\chi^4[2m_\chi^2 + 3m_{\chi_i^0}m_{\chi_j^0} + 2m_\chi(m_{\chi_i^0}+m_{\chi_j^0})] + $$
$$+ m_Z^4[3m_\chi^2 + 9m_{\chi_i^0}m_{\chi_j^0} + 5m_\chi(m_{\chi_i^0}+m_{\chi_j^0})]\,, \tag{5.56}$$

where $\Delta_{Zi} \equiv m_Z^2 - m_\chi^2 - m_{\chi_i^0}^2$. The couplings $C_{\pm i}^Z$ and $D_{\pm i}^Z$ are given in eqs. (4.95) and (4.96).

Higgs (h, H)-neutralino (χ_i^0) interference term:

$$\widetilde{a}_{ZZ}^{(h,H-\chi^0)} = 0\,, \tag{5.57}$$

$$\widetilde{b}_{ZZ}^{(h,H-\chi^0)} = \frac{1}{16\pi}\sum_{i=1}^{4}\mathrm{Re}\left[\left(\sum_{r=h,H}\frac{C^{ZZr}C_S^{\chi\chi r}}{s-m_r^2+i\Gamma_r m_r}\right)^*\right]\frac{1}{m_Z^4\Delta_{Zi}^2}C_S^{\chi\chi r}\times$$
$$\times\Big\{2m_\chi(m_\chi^2-m_Z^2)[-3m_Z^4 - 4m_\chi^3 m_{\chi_i^0} + 2m_Z^2 m_\chi(m_\chi+m_{\chi_i^0})] + \tag{5.58}$$
$$+\Delta_{Zi}\Big[-4m_\chi^4(2m_\chi+3m_\chi) + 2m_Z^2 m_\chi^2(5m_\chi+6m_{\chi_i^0}) - m_Z^4(5m_\chi+9m_{\chi_i^0})\Big]\Big\}.$$

5.6 $\chi\chi \to \bar{f}f$

This process involves the s-channel Higgs boson (h, H and A) and Z boson exchange and the t- and u-channel sfermion ($\widetilde{f}_a$) exchange

$$\widetilde{a}_{\bar{f}f} = \widetilde{a}_{\bar{f}f}^{(h,H)} + \widetilde{a}_{\bar{f}f}^{(A)} + \widetilde{a}_{\bar{f}f}^{(Z)} + \widetilde{a}_{\bar{f}f}^{(\widetilde{f})} + \widetilde{a}_{\bar{f}f}^{(h,H-\widetilde{f})} + \widetilde{a}_{\bar{f}f}^{(A-Z)} + \widetilde{a}_{\bar{f}f}^{(A-\widetilde{f})} + \widetilde{a}_{\bar{f}f}^{(Z-\widetilde{f})}\,, \tag{5.59}$$

$$\widetilde{b}_{\bar{f}f} = \widetilde{b}_{\bar{f}f}^{(h,H)} + \widetilde{b}_{\bar{f}f}^{(A)} + \widetilde{b}_{\bar{f}f}^{(Z)} + \widetilde{b}_{\bar{f}f}^{(\widetilde{f})} + \widetilde{b}_{\bar{f}f}^{(h,H-\widetilde{f})} + \widetilde{b}_{\bar{f}f}^{(A-Z)} + \widetilde{b}_{\bar{f}f}^{(A-\widetilde{f})} + \widetilde{b}_{\bar{f}f}^{(Z-\widetilde{f})}\;: \tag{5.60}$$

CP-even Higgs-boson (h, H) exchange:

$$\widetilde{a}_{\bar{f}f}^{(h,H)} = 0\,, \tag{5.61}$$

$$\widetilde{b}_{\bar{f}f}^{(h,H)} = \frac{3}{4\pi}\left|\sum_{r=h,H}\frac{C_S^{ffr}C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r}\right|^2 (m_\chi^2 - m_f^2)\,; \tag{5.62}$$

CP-odd Higgs-boson (A) exchange:

$$\widetilde{a}_{\bar{f}f}^{(A)} = \frac{1}{2\pi}\left|\frac{C_P^{ffA}C_P^{\chi\chi A}}{4m_\chi^2 - m_A^2 + i\Gamma_A m_A}\right|^2 m_\chi^2\,, \tag{5.63}$$

$$\widetilde{b}_{\bar{f}f}^{(A)} = \frac{3}{2\pi}\left|\frac{C_P^{ffA}C_P^{\chi\chi A}}{4m_\chi^2 - m_A^2 + i\Gamma_A m_A}\right|^2 \frac{m_\chi^2 m_A^2(m_A^2 - 4m_\chi^2 + \Gamma_A^2)}{(4m_\chi^2 - m_A^2)^2 + (\Gamma_A m_A)^2}\,; \tag{5.64}$$

Z-boson exchange:

$$\widetilde{a}_{\bar{f}f}^{(Z)} = \frac{1}{2\pi}\left|\frac{C_A^{ffZ}C_A^{\chi\chi Z}}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z}\right|^2 \frac{m_f^2(m_Z^2 - 4m_\chi^2)^2}{m_Z^4}\,, \tag{5.65}$$

$$\begin{aligned}\widetilde{b}_{\bar{f}f}^{(Z)} = \frac{1}{2\pi}&\left|\frac{C_A^{ffZ}C_A^{\chi\chi Z}}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z}\right|^2 \frac{1}{m_Z^2((4m_\chi^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2)} \times \\ &\times\Bigg[2|C_A^{ffZ}|^2\Big\{m_Z^2(m_\chi^2 - m_f^2)(m_Z^2 - 4m_\chi^2)^2 + \\ &\qquad + \Gamma_Z^2[m_\chi^2 m_Z^4 + m_f^2(24m_\chi^4 - 6m_Z^2 m_\chi^2 - m_Z^4)]\Big\} + \\ &\quad + m_Z^2|C_V^{ffZ}|^2\Big\{(2m_\chi^2 + m_f^2)[(4m_\chi^2 - m_Z^2)^2 + m_Z^2\Gamma_Z^2]\Big\}\Bigg]\,;\end{aligned} \tag{5.66}$$

Sfermion $(\widetilde{f}_a)$ exchange:

$$\widetilde{a}_{\bar{f}f}^{(\widetilde{f})} = \frac{1}{32\pi}\sum_{a,b}\frac{(m_f C_+^a + m_\chi D_+^a)(m_f C_+^b + m_\chi D_+^b)}{\Delta_{\widetilde{f}_a}\Delta_{\widetilde{f}_b}}\,, \tag{5.67}$$

$$\begin{aligned}\widetilde{b}_{\bar{f}f}^{(\widetilde{f})} = \frac{1}{64\pi}&\sum_{a,b}\frac{1}{\Delta_{\widetilde{f}_a}^3\Delta_{\widetilde{f}_b}^3} \times \\ \times\Big\{&C_+^a C_+^b\Big[8m_f^2 m_\chi^2(m_\chi^2 - m_f^2)\Delta_{\widetilde{f}_a}^2 - 4m_\chi^2(m_f^4 + m_\chi^2 m_f^2 - 2m_\chi^4)\Delta_{\widetilde{f}_a}\Delta_{\widetilde{f}_b} + \\ &\quad + 4m_\chi^2(m_f^2 + 2m_\chi^2)\Delta_{\widetilde{f}_a}^2\Delta_{\widetilde{f}_b} + 4(m_\chi^2 - m_f^2)\Delta_{\widetilde{f}_a}^2\Delta_{\widetilde{f}_b}^2\Big] + \\ &+ D_+^a D_+^b\Big[8m_\chi^4(m_\chi^2 - m_f^2)\Delta_{\widetilde{f}_a}^2 + 4m_\chi^2(m_\chi^4 + m_\chi^2 m_f^2 - 2m_f^4)\Delta_{\widetilde{f}_a}\Delta_{\widetilde{f}_b} + \\ &\quad + 4m_\chi^2(5m_\chi^2 - 2m_f^2)\Delta_{\widetilde{f}_a}^2\Delta_{\widetilde{f}_b} - 3(m_f^2 - 3m_\chi^2)\Delta_{\widetilde{f}_a}^2\Delta_{\widetilde{f}_b}^2\Big] + \\ &+ C_-^a C_-^b\Big[8m_\chi^2(m_\chi^2 - m_f^2)^2\Delta_{\widetilde{f}_a}\Delta_{\widetilde{f}_b} + 8m_\chi^2(m_\chi^2 - m_f^2)\Delta_{\widetilde{f}_a}^2\Delta_{\widetilde{f}_b} + \\ &\quad + 2(m_f^2 + 2m_\chi^2)\Delta_{\widetilde{f}_a}^2\Delta_{\widetilde{f}_b}^2\Big] +\end{aligned}$$

$$+ C_+^a D_+^b 2m_f m_\chi \Big[8m_\chi^2 (m_\chi^2 - m_f^2) \Delta_{\tilde{f}_a}^2 + 12 m_\chi^2 (m_\chi^2 - m_f^2) \Delta_{\tilde{f}_a} \Delta_{\tilde{f}_b} + 3\Delta_{\tilde{f}_a}^2 \Delta_{\tilde{f}_b}^2 - $$
$$- 2(m_f^2 - 4m_\chi^2) \Delta_{\tilde{f}_a} \Delta_{\tilde{f}_b}^2 - 6(m_f^2 - 2m_\chi^2) \Delta_{\tilde{f}_a}^2 \Delta_{\tilde{f}_b} \Big] \Big\}, \qquad (5.68)$$

where $\Delta_{\tilde{f}_a} \equiv m_f^2 - m_\chi^2 - m_{\tilde{f}_a}^2$. The index a counts sfermions so that $a = 1, \ldots, 6$ for squarks and charged sleptons, and $a = 1, 2, 3$ for sneutrinos. The couplings $C_\pm^a$ and $D_\pm^a$ are given in eqs. (4.105) and (4.106).

Higgs (h, H)-sfermion $(\tilde{f}_a)$ interference term:

$$\tilde{a}_{\bar{f}f}^{(h,H-\tilde{f})} = 0 \,, \qquad (5.69)$$
$$\tilde{b}_{\bar{f}f}^{(h,H-\tilde{f})} = -\frac{1}{8\pi} \sum_a \mathrm{Re} \left[\sum_{r=h,H} \frac{C_S^{ffr} C_S^{\chi\chi r}}{4m_\chi^2 - m_r^2 + i\Gamma_r m_r} \right] \frac{(m_\chi^2 - m_f^2)}{\Delta_{\tilde{f}_a}^2} \times$$
$$\times \left[C_+^a 2m_f m_\chi + D_+^a (2m_\chi^2 + 3\Delta_{\tilde{f}_a}) \right]; \qquad (5.70)$$

Higgs (A)-Z interference term:

$$\tilde{a}_{\bar{f}f}^{(A-Z)} = \frac{1}{\pi} \mathrm{Re} \left[\left(\frac{C_P^{ffA} C_P^{\chi\chi A}}{4m_\chi^2 - m_A^2 + i\Gamma_A m_A} \right)^* \left(\frac{C_A^{ffZ} C_A^{\chi\chi Z}}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z} \right) \right] \times$$
$$\times \frac{m_\chi m_f (m_Z^2 - 4m_\chi^2)}{m_Z^2} \,, \qquad (5.71)$$
$$\tilde{b}_{\bar{f}f}^{(A-Z)} = \frac{3}{2\pi} \mathrm{Re} \Bigg[\left(\frac{C_P^{ffA} C_P^{\chi\chi A}}{4m_\chi^2 - m_A^2 + i\Gamma_A m_A} \right)^* \left(\frac{C_A^{ffZ} C_A^{\chi\chi Z}}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z} \right)$$
$$\times \frac{m_\chi m_f}{m_Z^2} \Big[m_A (m_Z^2 - 4m_\chi^2)^2 (m_A + i\Gamma_A) + \qquad (5.72)$$
$$+ m_Z \Gamma_Z \Big\{ m_A \Gamma_A (m_Z^2 - 8m_\chi^2) - i[16 m_\chi^4 + m_A^2 (m_Z^2 - 8m_\chi^2)] \Big\} \Big] \Bigg];$$

Higgs (A)-sfermion $(\tilde{f}_a)$ interference term:

$$\tilde{a}_{\bar{f}f}^{(A-\tilde{f})} = -\frac{1}{4\pi} \sum_a \mathrm{Re} \left[\frac{C_P^{ffA} C_P^{\chi\chi A}}{4m_\chi^2 - m_A^2 + i\Gamma_A m_A} \right] \frac{(m_f C_+^a + m_\chi D_+^a)}{\Delta_{\tilde{f}_a}} \,, \qquad (5.73)$$
$$\tilde{b}_{\bar{f}f}^{(A-\tilde{f})} = -\frac{1}{8\pi} \sum_a \mathrm{Re} \Bigg[\left(\frac{C_P^{ffA} C_P^{\chi\chi A}}{(4m_\chi^2 - m_A^2 + i\Gamma_A m_A)^2} \right) \frac{m_\chi}{\Delta_{\tilde{f}_a}^3} \times$$
$$\times \Big\{ C_+^a \Big[4m_f m_\chi^2 (m_\chi^2 - m_f^2) P_A + 6 m_f m_\chi^2 P_A \Delta_{\tilde{f}_a} -$$
$$- 3 m_f m_A (m_A - i\Gamma_A) \Delta_{\tilde{f}_a}^2 \Big] +$$
$$+ D_+^a \Big[4m_\chi^3 (m_\chi^2 - m_f^2) P_A - 2m_\chi (m_f^2 - 4m_\chi^2) P_A \Delta_{\tilde{f}_a} +$$
$$+ 6 m_\chi (2m_\chi^2 - m_A^2 + i\Gamma_A m_A) \Delta_{\tilde{f}_a}^2 \Big] \Big\} \Bigg]; \qquad (5.74)$$

where $P_A \equiv 4m_\chi^2 - m_A^2 + i\Gamma_A m_A$.

Z-sfermion ($\widetilde{f}_a$) interference term:

$$
\begin{aligned}
\widetilde{a}_{\bar{f}f}^{(Z-\widetilde{f})} &= -\frac{1}{4\pi}\sum_a \mathrm{Re}\left[\frac{C_A^{ffZ}C_A^{\chi\chi Z}}{4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z}\right]\frac{m_f(m_Z^2 - 4m_\chi^2)}{m_Z^2}\times \\
&\quad \times \frac{(m_f C_+^a + m_\chi D_+^a)}{\Delta_{\widetilde{f}_a}},
\end{aligned}
\tag{5.75}
$$

$$
\begin{aligned}
\widetilde{b}_{\bar{f}f}^{(Z-\widetilde{f})} &= -\frac{1}{8\pi}\sum_a \mathrm{Re}\Bigg[\left(\frac{C_A^{\chi\chi Z}}{(4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z)^2}\right)\frac{1}{m_Z^2\Delta_{\widetilde{f}_a}^3}\times \\
&\qquad \times\Big[C_V^{ffZ}C_-^a\Big\{2m_Z^2 P_Z\Delta_{\widetilde{f}_a}[2m_\chi^2(m_\chi^2 + \Delta_{\widetilde{f}_a}) + m_f^2(-2m_\chi^2 + \Delta_{\widetilde{f}_a})]\Big\} + \\
&\qquad + C_A^{ffZ}\Big\{C_+^a\Big[2m_f^2 m_\chi^2(m_\chi^2 - m_f^2)(m_Z^2 - 4m_\chi^2)P_Z + \\
&\qquad\quad + m_\chi^2[m_f^2 m_Z^2 + 2m_\chi^2(m_Z^2 - 6m_f^2)]P_Z\Delta_{\widetilde{f}_a} + \\
&\qquad\quad + 2m_Z\{-m_Z(m_\chi^2 - m_f^2)(m_Z^2 - 4m_\chi^2) + \\
&\qquad\quad + i\Gamma_Z[m_Z^2 m_\chi^2 - m_f^2(m_Z^2 + 3m_\chi^2)]\}\Delta_{\widetilde{f}_a}^2\Big] + \\
&\qquad + m_f m_\chi D_+^a\Big[4m_\chi^2(m_\chi^2 - m_f^2)(m_Z^2 - 4m_\chi^2)P_Z + \\
&\qquad + 2[6m_Z^2 m_\chi^2 - 16m_\chi^4 - m_f^2(3m_Z^2 - 4m_\chi^2)]P_Z\Delta_{\widetilde{f}_a} - \\
&\qquad - 3[(m_Z^2 - 4m_\chi^2)^2 - i m_Z\Gamma_Z(m_Z^2 - 8m_\chi^2)]\Delta_{\widetilde{f}_a}^2\Big]\Big\}\Big]\Bigg];
\end{aligned}
\tag{5.76}
$$

where $P_Z \equiv 4m_\chi^2 - m_Z^2 + i\Gamma_Z m_Z$.

6. Summary

The neutralino is undoubtedly the most popular candidate for a WIMP dark matter in the Universe. Supersymmetry remains arguably the most promising extension of the Standard Model. The next several years will witness extensive searches for supersymmetry in colliders as well as for WIMPs in underground detectors. Measurements of the cosmological parameters, and in particular of the relic abundance of the dark matter, have already reached the accuracy of a few per cent and more progress is expected.

In light of this, theoretical computations of the neutralino relic abundance need to be now performed with at least the same, if not better, level of precision, if one wants to reliably compare theoretical predictions with observations. Motivated by this goal, we have derived a full set of exact, analytic expressions for the neutralino pair-annihilation cross sections into all tree-level two-body final states in the framework of the MSSM.

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A. The MSSM lagrangian

In this appendix, the MSSM lagrangian is given explicitly in the mass eigenstates, which makes it easier to read out relevant Feynman rules.

Z-boson-fermion-fermion.

$$\mathcal{L} = \sum_f \bar{f}\gamma^\mu \left[C_V^{ffZ} - C_A^{ffZ}\gamma_5\right] f Z_\mu \,, \tag{A.1}$$

where

$$C_V^{ffZ} = -\frac{g}{2\cos\theta_W}(T_{3f} - 2\sin^2\theta_W Q_f)\,, \tag{A.2}$$

$$C_A^{ffZ} = -\frac{g}{2\cos\theta_W}T_{3f}\,, \tag{A.3}$$

and g is the usual gauge coupling of $\mathrm{SU}(2)_L$ whereas Q_f and T_{3f} were defined below eq. (3.6).

W-chargino-neutralino.

$$\mathcal{L} = \sum_{k=1}^{2}\sum_{i=1}^{4} \overline{\chi_k^-}\gamma^\mu \left[C_V^{\chi_k^+\chi_i^0 W^-} - C_A^{\chi_k^+\chi_i^0 W^-}\gamma_5\right]\chi_i^0 W_\mu^- + \text{h.c.}\,, \tag{A.4}$$

where

$$C_V^{\chi_k^+\chi_i^0 W^-} = -\frac{1}{2}g(O_{ik}^L + O_{ik}^R)\,, \tag{A.5}$$

$$C_A^{\chi_k^+\chi_i^0 W^-} = \frac{1}{2}g(O_{ik}^L - O_{ik}^R)\,, \tag{A.6}$$

and

$$O_{ik}^L = -\sqrt{\frac{1}{2}}N_{i4}V_{k2}^* + N_{i2}V_{k1}^*\,, \tag{A.7}$$

$$O_{ik}^R = \sqrt{\frac{1}{2}}N_{i3}^*U_{k2} + N_{i2}^*U_{k1}^*\,, \tag{A.8}$$

where the matrices N, U, V were defined via eqs. (3.3) and (3.5).

Higgs-chargino-neutralino.

$$\mathcal{L} = \sum_{k=1}^{2}\sum_{i=1}^{4} \overline{\chi_k^-}\left[C_S^{\chi_k^+\chi_i^0 H^-} - C_P^{\chi_k^+\chi_i^0 H^-}\gamma_5\right]\chi_i^0 H^- + \text{h.c.}\,, \tag{A.9}$$

where

$$C_S^{\chi_k^+\chi_i^0 H^-} = -\frac{1}{2}(Q_{ik}^{\prime L} + Q_{ik}^{\prime R})\,, \tag{A.10}$$

$$C_P^{\chi_k^+\chi_i^0 H^-} = -\frac{1}{2}(Q_{ik}^{\prime L} - Q_{ik}^{\prime R})\,, \tag{A.11}$$

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and

$$Q'^{L}_{ik} = g\cos\beta\left[N^*_{i4}V^*_{k1} + \sqrt{\frac{1}{2}}(N^*_{i2} + N^*_{i1}\tan\theta_W)V^*_{k2}\right], \tag{A.12}$$

$$Q'^{R}_{ik} = g\sin\beta\left[N_{i3}U_{k1} - \sqrt{\frac{1}{2}}(N_{i2} + N_{i1}\tan\theta_W)U_{k2}\right]. \tag{A.13}$$

Z-neutralino-neutralino.

$$\mathcal{L} = \frac{1}{2}\sum_{i,j=1}^{4}\overline{\chi^0_i}\gamma^\mu\left[C_V^{\chi^0_i\chi^0_j Z} - C_A^{\chi^0_i\chi^0_j Z}\gamma_5\right]\chi^0_j Z_\mu\,, \tag{A.14}$$

where

$$C_V^{\chi^0_i\chi^0_j Z} = \frac{g}{2\cos\theta_W}(O''^{L}_{ij} - O''^{L*}_{ij})\,, \tag{A.15}$$

$$C_A^{\chi^0_i\chi^0_j Z} = \frac{g}{2\cos\theta_W}(O''^{L}_{ij} + O''^{L*}_{ij})\,, \tag{A.16}$$

and

$$O''^{L}_{ij} = \frac{1}{2}(-N_{i3}N^*_{j3} + N_{i4}N^*_{j4})\,. \tag{A.17}$$

We neglect CP violation hence $C_V^{\chi^0_i\chi^0_j Z} = 0$.

Gauge-Higgs-Higgs.

$$\begin{aligned}\mathcal{L} = i&\left[C^{hAZ}(A\overleftrightarrow{\partial}^{\mu}h) + C^{HAZ}(A\overleftrightarrow{\partial}^{\mu}H) + C^{H^+H^-Z}(H^+\overleftrightarrow{\partial}^{\mu}H^-)\right]Z_\mu + \\ &+\left\{iW^-_\mu\left[C^{W^-H^+h}(h\overleftrightarrow{\partial}^{\mu}H^+) + C^{W^-H^+H}(H\overleftrightarrow{\partial}^{\mu}H^+) + C^{W^-H^+A}(A\overleftrightarrow{\partial}^{\mu}H^+)\right] + \text{h.c.}\right\},\end{aligned} \tag{A.18}$$

where

$$C^{hAZ} = -\frac{ig}{2\cos\theta_W}\cos(\alpha-\beta)\,, \tag{A.19}$$

$$C^{HAZ} = -\frac{ig}{2\cos\theta_W}\sin(\alpha-\beta)\,, \tag{A.20}$$

$$C^{H^+H^-Z} = \frac{g}{2\cos\theta_W}(1 - 2\sin^2\theta_W)\,, \tag{A.21}$$

$$C^{W^-H^+h} = -\frac{g}{2}\cos(\alpha-\beta)\,, \tag{A.22}$$

$$C^{W^-H^+H} = -\frac{g}{2}\sin(\alpha-\beta)\,, \tag{A.23}$$

$$C^{W^-H^+A} = \frac{ig}{2}\,. \tag{A.24}$$

Higgs-fermion-fermion.

$$\mathcal{L} = \sum_f \left[C_S^{ffh}\bar{f}fh + C_S^{ffH}\bar{f}fH + C_P^{ffA}\bar{f}\gamma_5 fA\right] \tag{A.25}$$

$$\begin{aligned}\equiv \sum_{n=1}^{3}\Bigg[&\frac{gm_{e_n}\sin\alpha}{2m_W\cos\beta}\bar{e}_n e_n h - \frac{gm_{e_n}\cos\alpha}{2m_W\cos\beta}\bar{e}_n e_n H + i\frac{gm_{e_n}}{2m_W}\tan\beta\bar{e}_n\gamma_5 e_n A -\\ &-\frac{gm_{u_n}\cos\alpha}{2m_W\sin\beta}\bar{u}_n u_n h - \frac{gm_{u_n}\sin\alpha}{2m_W\sin\beta}\bar{u}_n u_n H + i\frac{gm_{u_n}}{2m_W}\cot\beta\bar{u}_n\gamma_5 u_n A +\\ &+\frac{gm_{d_n}\sin\alpha}{2m_W\cos\beta}\bar{d}_n d_n h - \frac{gm_{d_n}\cos\alpha}{2m_W\cos\beta}\bar{d}_n d_n H + i\frac{gm_{d_n}}{2m_W}\tan\beta\bar{d}_n\gamma_5 d_n A\Bigg],\end{aligned} \tag{A.26}$$

where $n = 1, 2, 3$ is an index for generation, so $e_n = e, \mu, \tau$, etc.

Higgs-gauge-gauge.

$$\mathcal{L} = \left[C^{W^+W^-h}h + C^{W^+W^-H}H\right]W_\mu^+W^{-\mu} + \frac{1}{2}\left[C^{ZZh}h + C^{ZZH}H\right]Z_\mu Z^\mu, \tag{A.27}$$

where

$$C^{W^+W^-h} = -gm_W\sin(\alpha-\beta), \tag{A.28}$$

$$C^{W^+W^-H} = gm_W\cos(\alpha-\beta), \tag{A.29}$$

$$C^{ZZh} = -\frac{gm_Z}{\cos\theta_W}\sin(\alpha-\beta), \tag{A.30}$$

$$C^{ZZH} = \frac{gm_Z}{\cos\theta_W}\cos(\alpha-\beta). \tag{A.31}$$

Higgs-neutralino-neutralino.

$$\mathcal{L} = \frac{1}{2}\sum_{i,j=1}^{4}\overline{\chi_i^0}\left[C_S^{\chi_i^0\chi_j^0 H}H + C_S^{\chi_i^0\chi_j^0 h}h + C_P^{\chi_i^0\chi_j^0 A}\gamma_5 A\right]\chi_j^0, \tag{A.32}$$

where

$$C_S^{\chi_i^0\chi_j^0 H} = -\frac{g}{2}\{[N_{i2} - N_{i1}\tan\theta_W][\cos\alpha N_{j3} - \sin\alpha N_{j4}] + (i\leftrightarrow j)\}, \tag{A.33}$$

$$C_S^{\chi_i^0\chi_j^0 h} = -\frac{g}{2}\{[N_{i2} - N_{i1}\tan\theta_W][-\sin\alpha N_{j3} - \cos\alpha N_{j4}] + (i\leftrightarrow j)\}, \tag{A.34}$$

$$C_P^{\chi_i^0\chi_j^0 A} = -\frac{ig}{2}\{[N_{i2} - N_{i1}\tan\theta_W][\sin\beta N_{j3} - \cos\beta N_{j4}] + (i\leftrightarrow j)\}. \tag{A.35}$$

Note that we assume no CP violating phases in the neutralino mass matrix and we use the convention that N_{ij} is real.

Gauge-gauge-gauge.

$$\begin{aligned}\mathcal{L} = iC^{WWZ}\Big[&(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-)W^{+\mu}Z^\nu - (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)W^{-\mu}Z^\nu -\\ &-(\partial_\mu Z_\nu - \partial_\nu Z_\mu)W^{+\mu}W^{-\nu}\Big],\end{aligned} \tag{A.36}$$

where

$$C^{WWZ} = g\cos\theta_W. \tag{A.37}$$

Neutral Higgs-Higgs-Higgs.

$$\begin{aligned}\mathcal{L} = & \frac{1}{6}C^{hhh}h^3 + \frac{1}{6}C^{HHH}H^3 + \frac{1}{2}C^{hhH}h^2H + \frac{1}{2}C^{HHh}H^2h + \\ & + \frac{1}{2}(C^{AAh}h + C^{AAH}H)A^2\,, \end{aligned} \tag{A.38}$$

where

$$C^{hhh} = -\frac{3g}{2\cos\theta_W}m_Z\cos 2\alpha\sin(\alpha+\beta)\,, \tag{A.39}$$

$$C^{HHH} = -\frac{3g}{2\cos\theta_W}m_Z\cos 2\alpha\cos(\alpha+\beta)\,, \tag{A.40}$$

$$C^{AAh} = -\frac{g}{2\cos\theta_W}m_Z\cos 2\beta\sin(\alpha+\beta)\,, \tag{A.41}$$

$$C^{AAH} = \frac{g}{2\cos\theta_W}m_Z\cos 2\beta\cos(\alpha+\beta)\,, \tag{A.42}$$

$$C^{hhH} = -\frac{g}{2\cos\theta_W}m_Z[2\sin 2\alpha\sin(\alpha+\beta) - \cos 2\alpha\cos(\alpha+\beta)]\,, \tag{A.43}$$

$$C^{HHh} = \frac{g}{2\cos\theta_W}m_Z[2\sin 2\alpha\cos(\alpha+\beta) + \cos 2\alpha\sin(\alpha+\beta)]\,. \tag{A.44}$$

$\mathbf{H^+}$-$\mathbf{H^-}$-Higgs.

$$\mathcal{L} = (C^{H^+H^-h}h + C^{H^+H^-H}H)H^+H^-\,, \tag{A.45}$$

where

$$C^{H^+H^-h} = gm_Z\left[\cos\theta_W\sin(\alpha-\beta) - \frac{1}{2\cos\theta_W}\sin(\alpha+\beta)\cos 2\beta\right], \tag{A.46}$$

$$C^{H^+H^-H} = -gm_Z\left[\cos\theta_W\cos(\alpha-\beta) - \frac{1}{2\cos\theta_W}\cos(\alpha+\beta)\cos 2\beta\right]. \tag{A.47}$$

Neutralino-fermion-sfermion.

$$\begin{aligned}\mathcal{L} = \sum_{i=1}^{4}\sum_{n=1}^{3}\Bigg\{ & \sum_{f=u,d,e}\sum_{a=1}^{6}\overline{\chi_i^0}\left(\Lambda^{(f)L}_{nai}\frac{1-\gamma_5}{2} + \Lambda^{(f)R}_{nai}\frac{1+\gamma_5}{2}\right)f_n\widetilde{f}^*_a + \\ & + \sum_{a=1}^{3}\overline{\chi_i^0}\Lambda^{(\nu)L}_{nai}\frac{1-\gamma_5}{2}\nu_n\tilde{\nu}^*_a + \text{h.c.}\Bigg\}\,, \end{aligned} \tag{A.48}$$

where

$$\Lambda^{(f)L}_{nai} = -\frac{g}{\sqrt{2}}\left(\Gamma^{an}_{(f)L}\ell^{\chi_i^0 f_n L} + \Gamma^{an}_{(f)R}\ell^{\chi_i^0 f_n R}\right), \tag{A.49}$$

$$\Lambda^{(f)R}_{nai} = -\frac{g}{\sqrt{2}}\left(\Gamma^{an}_{(f)L}r^{\chi_i^0 f_n L} + \Gamma^{an}_{(f)R}r^{\chi_i^0 f_n R}\right). \tag{A.50}$$

Note that in eqs. (A.49) and (A.50), $f = u, d, e, \nu$ represents the type of the fermion, while f in the main text denotes each fermion $f = u, c, t$, etc. The slepton and squark mass eigenstates $\widetilde{f}_a$ ($\tilde{\nu}_a$ with $a = 1,2,3$ and $\tilde{e}_a$, $\tilde{u}_a$ and $\tilde{d}_a$ with $a = 1,\ldots,6$) are related to the sfermion gauge eigenstates $\widetilde{f}_{nL}$ and $\widetilde{f}_{nR}$ ($n = 1,2,3$) via

$$\widetilde{f}_{nL} = \sum_a \widetilde{f}_a \Gamma^{an*}_{(f)L}\,, \tag{A.51}$$

$$\widetilde{f}_{nR} = \sum_a \widetilde{f}_a \Gamma^{an*}_{(f)R}\,, \tag{A.52}$$

where $\widetilde{V}_f = (\Gamma_{(f)L}, \Gamma_{(f)R})$ denotes a 6×6 matrix which diagonalizes the sfermions mass matrix given by eqs. (3.7)–(3.10): $\widetilde{V}_f\ \mathcal{M}^2_{\widetilde{f}}\ \widetilde{V}^\dagger_f = \mathrm{diag}(m^2_{\widetilde{f}_1},\ldots,m^2_{\widetilde{f}_6})$. Note that in the case of squarks and charged sleptons the mixing matrices $\Gamma_{(u)L,R}, \Gamma_{(d)L,R}$ and $\Gamma_{(e)L,R}$ have dimension 6×3, while for sneutrinos $\Gamma_{(\nu)L}$ is a mixing matrix of order 3×3. (We neglect here the CKM matrix for simplicity.) Finally $\ell^{\chi^0_i f_n L,R}$ and $r^{\chi^0_i f_n L,R}$ are given as follows:

$$\begin{aligned}
\ell^{\chi^0_i \nu_n L} &= N_{i2} - \tan\theta_W N_{i1}\,, &(A.53)\\
\ell^{\chi^0_i e_n L} &= -N_{i2} - \tan\theta_W N_{i1}\,, &(A.54)\\
\ell^{\chi^0_i u_n L} &= N_{i2} + \frac{1}{3}\tan\theta_W N_{i1}\,, &(A.55)\\
\ell^{\chi^0_i d_n L} &= -N_{i2} + \frac{1}{3}\tan\theta_W N_{i1}\,, &(A.56)\\
r^{\chi^0_i \nu_n L} &= 0\,, &(A.57)\\
r^{\chi^0_i e_n L} &= \frac{m_{e_n}}{m_W\cos\beta} N_{i3}\,, &(A.58)\\
r^{\chi^0_i u_n L} &= \frac{m_{u_n}}{m_W\sin\beta} N_{i4}\,, &(A.59)\\
r^{\chi^0_i d_n L} &= \frac{m_{d_n}}{m_W\cos\beta} N_{i3}\,, &(A.60)\\
\ell^{\chi^0_i \nu_n R} &= 0\,, &(A.61)\\
\ell^{\chi^0_i e_n R} &= \frac{m_{e_n}}{m_W\cos\beta} N_{i3}\,, &(A.62)\\
\ell^{\chi^0_i u_n R} &= \frac{m_{u_n}}{m_W\sin\beta} N_{i4}\,, &(A.63)\\
\ell^{\chi^0_i d_n R} &= \frac{m_{d_n}}{m_W\cos\beta} N_{i3}\,, &(A.64)\\
r^{\chi^0_i \nu_n R} &= 0\,, &(A.65)\\
r^{\chi^0_i e_n R} &= 2\tan\theta_W N_{i1}\,, &(A.66)\\
r^{\chi^0_i u_n R} &= -\frac{4}{3}\tan\theta_W N_{i1}\,, &(A.67)\\
r^{\chi^0_i d_n R} &= \frac{2}{3}\tan\theta_W N_{i1}\,. &(A.68)
\end{aligned}$$

B. Auxiliary functions

Here we give expressions for the auxiliary functions used in the text. First, we define

$$D(s,x,y_1,y_2) \equiv x + \frac{y_1+y_2}{2} - \frac{s}{2}\,, \tag{B.1}$$

$$F(s,x,y_1,y_2) \equiv \frac{1}{2}\sqrt{s-4x}\sqrt{s-(\sqrt{y_1}+\sqrt{y_2})^2}\sqrt{1-\frac{(\sqrt{y_1}-\sqrt{y_2})^2}{s}}\,. \tag{B.2}$$

If we define $D \equiv D(s,x,y_1,y_2)$, $F \equiv F(s,x,y_1,y_2)$, $t_\pm(s,x,y_1,y_2) \equiv D \pm F$ and $(\mathcal{T}_i, \mathcal{Y}_i) \equiv (\mathcal{T}_i, \mathcal{Y}_i)(s,x,y_1,y_2,z_1,z_2)$ $(i = 0,\ldots,4)$, then

$$\mathcal{F}(s,x,y_1,y_2,z) = \frac{1}{2F} \ln \left| \frac{t_+(s,x,y_1,y_2) - z}{t_-(s,x,y_1,y_2) - z} \right|, \tag{B.3}$$

and

$$\mathcal{T}_0 = \frac{1}{z_1 - z_2} \left[\mathcal{F}(s,x,y_1,y_2,z_1) - \mathcal{F}(s,x,y_1,y_2,z_2) \right], \tag{B.4}$$

$$\mathcal{T}_1 = \frac{1}{z_1 - z_2} \left[z_1\mathcal{F}(s,x,y_1,y_2,z_1) - z_2\mathcal{F}(s,x,y_1,y_2,z_2) \right], \tag{B.5}$$

$$\mathcal{T}_2 = 1 + \frac{1}{z_1 - z_2} \left[z_1^2\mathcal{F}(s,x,y_1,y_2,z_1) - z_2^2\mathcal{F}(s,x,y_1,y_2,z_2) \right], \tag{B.6}$$

$$\mathcal{T}_3 = D + z_1 + z_2 + \frac{1}{z_1 - z_2} \left[z_1^3\mathcal{F}(s,x,y_1,y_2,z_1) - z_2^3\mathcal{F}(s,x,y_1,y_2,z_2) \right], \tag{B.7}$$

$$\begin{aligned} \mathcal{T}_4 &= D^2 + D(z_1 + z_2) + (z_1^2 + z_2^2 + z_1 z_2) + \frac{1}{3}F^2 + \\ &\quad + \frac{1}{z_1 - z_2} \left[z_1^4\mathcal{F}(s,x,y_1,y_2,z_1) - z_2^4\mathcal{F}(s,x,y_1,y_2,z_2) \right], \end{aligned} \tag{B.8}$$

$$\mathcal{Y}_0 = \frac{1}{z_1 + z_2 - 2D} \left[\mathcal{F}(s,x,y_1,y_2,z_1) + \mathcal{F}(s,x,y_1,y_2,z_2) \right], \tag{B.9}$$

$$\mathcal{Y}_1 = \frac{1}{z_1 + z_2 - 2D} \left[2(z_1 - D)\mathcal{F}(s,x,y_1,y_2,z_1) - 2(z_2 - D)\mathcal{F}(s,x,y_1,y_2,z_2) \right], \tag{B.10}$$

$$\begin{aligned} \mathcal{Y}_2 &= 1 + \frac{1}{z_1 + z_2 - 2D} \Big[z_1(z_1 - 2D)\mathcal{F}(s,x,y_1,y_2,z_1) + \\ &\qquad + z_2(z_2 - 2D)\mathcal{F}(s,x,y_1,y_2,z_2) \Big], \end{aligned} \tag{B.11}$$

$$\begin{aligned} \mathcal{Y}_3 &= 2(z_1 - z_2) + \frac{1}{z_1 + z_2 - 2D} \Big[z_1(2z_1^2 - 6z_1 D + 4D^2)\mathcal{F}(s,x,y_1,y_2,z_1) - \\ &\qquad - z_2(2z_2^2 - 6z_2 D + 4D^2)\mathcal{F}(s,x,y_1,y_2,z_2) \Big], \end{aligned} \tag{B.12}$$

$$\begin{aligned} \mathcal{Y}_4 &= -D^2 - D(z_1 + z_2) + (z_1^2 + z_2^2 - z_1 z_2) + \frac{1}{3}F^2 + \\ &\quad + \frac{1}{z_1 + z_2 - 2D} \Big[z_1^2(z_1^2 - 4z_1 D + 4D^2)\mathcal{F}(s,x,y_1,y_2,z_1) + \\ &\qquad + z_2^2(z_2^2 - 4z_2 D + 4D^2)\mathcal{F}(s,x,y_1,y_2,z_2) \Big]. \end{aligned} \tag{B.13}$$

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Erratum

- In eq. (4.30), the right-hand side should be multiplied with the factor $1/m_Z^2$.
- In eq. (4.59), the last term $(m_h^2 - m_Z^2)s$ should read $(m_h^2 - m_Z^2)^2 s$.
- In the fourth line of eq. (4.62), the term '$- - 2\mathcal{Y}_2$' should be '$-2\mathcal{Y}_2$'.
- In eq. (4.67), the right-hand side should be multiplied with the factor $1/m_Z^2$.
- In eq. (4.75), the right-hand side should be multiplied with the factor $1/m_Z^2$.
- In eq. (4.88), the right-hand side should be multiplied with the factor $1/m_W^4$.
- In the second line of eq. (4.94), the term '$-(m_Z^2 - m_\chi^2)^2(2m_Z^2 + m_\chi^2)\mathcal{T}_0$' should be '$-(m_Z^2 - m_\chi^2)^2(2m_Z^2 + m_\chi^2)^2\mathcal{T}_0$.'
- In eq. (5.41), the left-hand side '$\widetilde{a}_{WW}^{\chi^\pm}$' should read '$\widetilde{a}_{WW}^{(\chi^\pm)}$'.
- In eq. (5.42), the left-hand side '$\widetilde{b}_{WW}^{\chi^\pm}$' should read '$\widetilde{b}_{WW}^{(\chi^\pm)}$'.

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